場の量子論 p97check4.8

谷脇 貞善

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1　ディラック方程式

1.1　角運動量保存

$$i\frac{\partial}{\partial t}\psi = H\psi, \quad H = -i\gamma^0\sum_{j=1}^{3}\gamma^j\frac{\partial}{\partial x^j} + m\gamma^0 \tag{1}$$

$$\left[\frac{\partial}{\partial x^j}, \frac{\partial}{\partial x^k}\right] = 0 \tag{2}$$

$$\left[x^j, \frac{\partial}{\partial x^k}\right] = -\delta^j_k \tag{3}$$

$$[\gamma^2\gamma^3, \gamma^0\gamma^2] = \gamma^2\gamma^3\gamma^0\gamma^2 - \gamma^0\gamma^2\gamma^2\gamma^3 = \gamma^2\gamma^2\gamma^3\gamma^0 - \gamma^0\gamma^2\gamma^2\gamma^3 = -I_4\gamma^3\gamma^0 - \gamma^0(-I_4)\gamma^3 = 2\gamma^0\gamma^3 \tag{4}$$

$$[\gamma^2\gamma^3, \gamma^0\gamma^3] = \gamma^2\gamma^3\gamma^0\gamma^3 - \gamma^0\gamma^3\gamma^2\gamma^3 = -\gamma^2\gamma^0\gamma^3\gamma^3 + \gamma^0\gamma^2\gamma^3\gamma^3 = -\gamma^2\gamma^0(-I_4) + \gamma^0\gamma^2(-I_4) = -2\gamma^0\gamma^2 \tag{5}$$

$$L^1 = i\left\{x^2\frac{\partial}{\partial x^3} - x^3\frac{\partial}{\partial x^2}\right\} \tag{6}$$

$$S^1 = \frac{i}{2}\gamma^2\gamma^3 \tag{7}$$

$$[L^1, H] = \left[i\left\{x^2\frac{\partial}{\partial x^3} - x^3\frac{\partial}{\partial x^2}\right\}, -i\gamma^0\sum_{j=1}^{3}\gamma^j\frac{\partial}{\partial x^j} + m\gamma^0\right] \tag{8}$$

$$= \gamma^0\gamma^2\frac{\partial}{\partial x^3} - \gamma^0\gamma^3\frac{\partial}{\partial x^2} \tag{9}$$

$$[S^1, H] = \left[\frac{i}{2}\gamma^2\gamma^3, -i\gamma^0\sum_{j=1}^{3}\gamma^j\frac{\partial}{\partial x^j} + m\gamma^0\right] \tag{10}$$

$$= \frac{1}{2}\left[\gamma^2\gamma^3, \gamma^0\gamma^2\frac{\partial}{\partial x^2} + \gamma^0\gamma^3\frac{\partial}{\partial x^3}\right] \tag{11}$$

$$= \gamma^0\gamma^3\frac{\partial}{\partial x^2} - \gamma^0\gamma^2\frac{\partial}{\partial x^3} \tag{12}$$

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)