場の量子論 p102check4.9

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1　ディラック方程式

1.1　負エネルギー解

$$i\frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \left\{(\vec{\alpha}\cdot\vec{\pi}) + m_e\gamma^0 - eA^0 I_4\right\}\psi(t,\mathbf{x}) \tag{1}$$

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \quad \alpha_j = \gamma^0\gamma^j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}, \quad \vec{\pi} = -i\nabla + e\mathbf{A} \tag{2}$$

$$\psi(t,\mathbf{x}) = e^{-i(-m_e)t}\begin{pmatrix} \tilde{\varphi}(t,\mathbf{x}) \\ \tilde{\chi}(t,\mathbf{x}) \end{pmatrix} \tag{3}$$

$$e^{-i(-m_e)t}\begin{pmatrix} (i\frac{\partial}{\partial t} - m_e)I_2\tilde{\varphi}(t,\mathbf{x}) \\ (i\frac{\partial}{\partial t} - m_e)I_2\tilde{\chi}(t,\mathbf{x}) \end{pmatrix} = e^{-i(-m_e)t}\begin{pmatrix} (\vec{\sigma}\cdot\vec{\pi})\tilde{\chi}(t,\mathbf{x}) + (m_e - eA^0)I_2\tilde{\varphi}(t,\mathbf{x}) \\ (\vec{\sigma}\cdot\vec{\pi})\tilde{\varphi}(t,\mathbf{x}) + (-m_e - eA^0)I_2\tilde{\chi}(t,\mathbf{x}) \end{pmatrix} \tag{4}$$

$$i\frac{\partial}{\partial t}\tilde{\varphi}(t,\mathbf{x}) = (\vec{\sigma}\cdot\vec{\pi})\tilde{\chi}(t,\mathbf{x}) + (2m_e - eA^0)\tilde{\varphi}(t,\mathbf{x}) \tag{5}$$

$$0 = (\vec{\sigma}\cdot\vec{\pi})\tilde{\chi}(t,\mathbf{x}) + 2m_e\tilde{\varphi}(t,\mathbf{x}) \tag{6}$$

$$\tilde{\varphi}(t,\mathbf{x}) = -\frac{1}{2m_e}(\vec{\sigma}\cdot\vec{\pi})\tilde{\chi}(t,\mathbf{x}) \tag{7}$$

$$i\frac{\partial}{\partial t}\tilde{\chi}(t,\mathbf{x}) = (\vec{\sigma}\cdot\vec{\pi})\tilde{\varphi}(t,\mathbf{x}) - eA^0\tilde{\chi}(t,\mathbf{x}) \tag{8}$$

$$i\frac{\partial}{\partial t}\tilde{\chi}(t,\mathbf{x}) = \left\{-\frac{1}{2m_e}(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{\pi}) - eA^0\right\}\tilde{\chi}(t,\mathbf{x}) \tag{9}$$

$$i\frac{\partial}{\partial t}\tilde{\chi}(t,\mathbf{x}) = \left\{-\frac{1}{2m_e}\left[\vec{\pi}^2 + (\vec{\sigma}\cdot\mathbf{B})\right] - eA^0\right\}\tilde{\chi}(t,\mathbf{x}) \tag{10}$$

$$i\frac{\partial}{\partial t}\tilde{\chi}(t,\mathbf{x}) = \left\{-\frac{1}{2m_e}(-i\nabla + e\mathbf{A})^2 - \frac{(\vec{\sigma}\cdot\mathbf{B})}{2m_e} - eA^0\right\}\tilde{\chi}(t,\mathbf{x}) \tag{11}$$

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)