

場の量子論 p137check5.6

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1 ディラック方程式の相対論的構造

1.1 (5.47a)

$$(J_V^{\mu\nu})^\rho{}_\lambda = i(\eta^{\mu\rho}\delta^\nu{}_\lambda - \eta^{\nu\rho}\delta^\mu{}_\lambda) \quad (1)$$

$$(J_V^{01})^\rho{}_\lambda = i(\eta^{0\rho}\delta^1{}_\lambda - \eta^{1\rho}\delta^0{}_\lambda) = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2)$$

$$(J_V^{23})^\rho{}_\lambda = i(\eta^{2\rho}\delta^3{}_\lambda - \eta^{3\rho}\delta^2{}_\lambda) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \quad (3)$$

1.2 本題

$$f(0) = -i(\eta^{\mu\rho}J^{\nu\lambda} - \eta^{\nu\rho}J^{\mu\lambda} + \eta^{\mu\lambda}J^{\rho\nu} - \eta^{\nu\lambda}J^{\rho\mu}) \quad (4)$$

$$f(\mu \leftrightarrow \nu) = -i(\eta^{\nu\rho}J^{\mu\lambda} - \eta^{\mu\rho}J^{\nu\lambda} + \eta^{\nu\lambda}J^{\rho\mu} - \eta^{\mu\lambda}J^{\rho\nu}) \quad (5)$$

$$= -i(-\eta^{\mu\rho}J^{\nu\lambda} + \eta^{\nu\rho}J^{\mu\lambda} - \eta^{\mu\lambda}J^{\rho\nu} + \eta^{\nu\lambda}J^{\rho\mu}) = -f(0) \quad (6)$$

$$f(\rho \leftrightarrow \lambda) = -i(\eta^{\mu\lambda}J^{\nu\rho} - \eta^{\nu\lambda}J^{\mu\rho} + \eta^{\mu\rho}J^{\lambda\nu} - \eta^{\nu\rho}J^{\lambda\mu}) \quad (7)$$

$$= -i(-\eta^{\mu\lambda}J^{\rho\nu} + \eta^{\nu\lambda}J^{\rho\mu} - \eta^{\mu\rho}J^{\nu\lambda} + \eta^{\nu\rho}J^{\mu\lambda}) \quad (8)$$

$$= -i(-\eta^{\mu\rho}J^{\nu\lambda} + \eta^{\nu\rho}J^{\mu\lambda} - \eta^{\mu\lambda}J^{\rho\nu} + \eta^{\nu\lambda}J^{\rho\mu}) = -f(0) \quad (9)$$

$$f((\mu\nu) \leftrightarrow (\rho\lambda)) = -i(\eta^{\rho\mu}J^{\lambda\nu} - \eta^{\lambda\mu}J^{\rho\nu} + \eta^{\rho\nu}J^{\mu\lambda} - \eta^{\lambda\nu}J^{\mu\rho}) \quad (10)$$

$$= -i(-\eta^{\mu\rho}J^{\nu\lambda} - \eta^{\mu\lambda}J^{\rho\nu} + \eta^{\nu\rho}J^{\mu\lambda} + \eta^{\nu\lambda}J^{\rho\mu}) \quad (11)$$

$$= -i(-\eta^{\mu\rho}J^{\nu\lambda} + \eta^{\nu\rho}J^{\mu\lambda} - \eta^{\mu\lambda}J^{\rho\nu} + \eta^{\nu\lambda}J^{\rho\mu}) = -f(0) \quad (12)$$

参考文献

- [1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)