場の量子論 p140check5.7

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1　本題

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\begin{aligned}
T^{\mu_1\mu_2\cdots\mu_n} &= \mathrm{tr}(\gamma^{\mu_1}\gamma^{\mu_2}\cdots\gamma^{\mu_n}) && (1)\\
&= \mathrm{tr}(\Lambda^{\mu_1}{}_{\nu_1}S\gamma^{\nu_1}S^{-1}\Lambda^{\mu_2}{}_{\nu_2}S\gamma^{\nu_2}S^{-1}\cdots\Lambda^{\mu_n}{}_{\nu_n}S\gamma^{\nu_n}S^{-1}) && (2)\\
&= \Lambda^{\mu_1}{}_{\nu_1}\Lambda^{\mu_2}{}_{\nu_2}\cdots\Lambda^{\mu_n}{}_{\nu_n}\mathrm{tr}(S\gamma^{\nu_1}S^{-1}S\gamma^{\nu_2}S^{-1}S\cdots S^{-1}S\gamma^{\nu_n}S^{-1}) && (3)\\
&= \Lambda^{\mu_1}{}_{\nu_1}\Lambda^{\mu_2}{}_{\nu_2}\cdots\Lambda^{\mu_n}{}_{\nu_n}\mathrm{tr}(S\gamma^{\nu_1}\gamma^{\nu_2}\cdots\gamma^{\nu_n}S^{-1}) && (4)\\
&= \Lambda^{\mu_1}{}_{\nu_1}\Lambda^{\mu_2}{}_{\nu_2}\cdots\Lambda^{\mu_n}{}_{\nu_n}\mathrm{tr}(\gamma^{\nu_1}\gamma^{\nu_2}\cdots\gamma^{\nu_n}S^{-1}S) && (5)\\
&= \Lambda^{\mu_1}{}_{\nu_1}\Lambda^{\mu_2}{}_{\nu_2}\cdots\Lambda^{\mu_n}{}_{\nu_n}\mathrm{tr}(\gamma^{\nu_1}\gamma^{\nu_2}\cdots\gamma^{\nu_n}) && (6)\\
&= \Lambda^{\mu_1}{}_{\nu_1}\Lambda^{\mu_2}{}_{\nu_2}\cdots\Lambda^{\mu_n}{}_{\nu_n}T^{\nu_1\nu_2\cdots\nu_n} && (7)\\
T^{\mu_1\mu_2} &= \mathrm{tr}(\gamma^{\mu_1}\gamma^{\mu_2}) && (8)\\
&= \frac{1}{2}\mathrm{tr}(\gamma^{\mu_1}\gamma^{\mu_2}+\gamma^{\mu_2}\gamma^{\mu_1}) && (9)\\
&= \eta^{\mu_1\mu_2}\mathrm{tr}I_4 && (10)\\
&= 4\eta^{\mu_1\mu_2} && (11)\\
T^{\mu_1\mu_2\cdots\mu_{2n+1}} &= \mathrm{tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}) && (12)\\
&= \mathrm{tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}\gamma^5\gamma^5) && (13)\\
&= (-1)^{2n+1}\mathrm{tr}(\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}\gamma^5) && (14)\\
&= (-1)^{2n+1}\mathrm{tr}(\gamma^5\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}) && (15)\\
&= -T^{\mu_1\mu_2\cdots\mu_{2n+1}} && (16)\\
T^{\mu_1\mu_2\cdots\mu_{2n+1}} &= 0 && (17)
\end{aligned}
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参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)