

場の量子論 p197check7.4

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2024 年 1 月 14 日

1 ヤコビの恒等式

$$0 = [T^a, [T^b, T^c]] + [T^b, [T^c, T^a]] + [T^c, [T^a, T^b]] \quad (1)$$

$$= [T^a, i f^{bcd} T^d] + [T^b, i f^{cad} T^d] + [T^c, i f^{abd} T^d] \quad (2)$$

$$= i f^{bcd} [T^a, T^d] + i f^{cad} [T^b, T^d] + i f^{abd} [T^c, T^d] \quad (3)$$

$$= -f^{bcd} f^{ade} T^e - f^{cad} f^{bde} T^e - f^{abd} f^{cde} T^e \quad (4)$$

$$= -(f^{bcd} f^{ade} + f^{cad} f^{bde} + f^{abd} f^{cde}) T^e \quad (5)$$

2 随伴表現

$$f^{bcd} f^{ade} + f^{cad} f^{bde} + f^{abd} f^{cde} = 0 \quad (6)$$

$$-i f^{cbd} i f^{dae} + i f^{cad} i f^{dbe} - i f^{abd} i f^{cde} = 0 \quad (7)$$

$$-(T_{\text{adj}}^b)^{cd} (T_{\text{adj}}^a)^{de} + (T_{\text{adj}}^a)^{cd} (T_{\text{adj}}^b)^{de} = i f^{abd} (T_{\text{adj}}^d)^{ce} \quad (8)$$

$$T_{\text{adj}}^a T_{\text{adj}}^b - T_{\text{adj}}^b T_{\text{adj}}^a = i f^{abd} T_{\text{adj}}^d \quad (9)$$

$$[T_{\text{adj}}^a, T_{\text{adj}}^b] = i f^{abd} T_{\text{adj}}^d \quad (10)$$

参考文献

- [1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)