場の量子論 p214check7.12

谷脇 貞善

2024 年 6 月 7 日

1　本文

$$\begin{pmatrix} i\gamma^\mu (D^u_\mu)^a{}_b & i\gamma^\mu \frac{ig_2}{\sqrt{2}} W^+_\mu \delta^a{}_b \\ i\gamma^\mu \frac{ig_2}{\sqrt{2}} W^-_\mu \delta^a{}_b & i\gamma^\mu (D^d_\mu)^a{}_b \end{pmatrix} \begin{pmatrix} u^b_L \\ d^b_L \end{pmatrix} = 0 \tag{1}$$

$$(D^u_\mu)^a{}_b = \partial_\mu \delta^a{}_b + ig_3 (G_\mu)^a{}_b + i\frac{g_2}{2} W^3_\mu \delta^a{}_b + i\frac{g_1}{2}\left(\frac{1}{3}\right) B_\mu \delta^a{}_b \tag{2}$$

$$(D^d_\mu)^a{}_b = \partial_\mu \delta^a{}_b + ig_3 (G_\mu)^a{}_b - i\frac{g_2}{2} W^3_\mu \delta^a{}_b + i\frac{g_1}{2}\left(\frac{1}{3}\right) B_\mu \delta^a{}_b \tag{3}$$

$$u_L \to {u'}_L = U_3(x) u_L \tag{4}$$

$$({D'}^u_\mu)^a{}_b {u'}^b_L = \left[\partial_\mu \delta^a{}_b + ig_3 ({G'}_\mu)^a{}_b + i\frac{g_2}{2} {W'}^3_\mu \delta^a{}_b + i\frac{g_1}{2}\left(\frac{1}{3}\right) {B'}_\mu \delta^a{}_b\right] {u'}^b_L \tag{5}$$

$$= \left[\partial_\mu \delta^a{}_b + ig_3 ({G'}_\mu)^a{}_b + i\frac{g_2}{2} {W'}^3_\mu \delta^a{}_b + i\frac{g_1}{2}\left(\frac{1}{3}\right) {B'}_\mu \delta^a{}_b\right] (U_3)^b{}_c u^c_L \tag{6}$$

$$\begin{aligned} &= \Big\{(U_3)^a{}_b \partial_\mu \delta^b{}_c + [\partial_\mu (U_3)^a{}_c] + ig_3 ({G'}_\mu)^a{}_b (U_3)^b{}_c + (U_3)^a{}_b i\frac{g_2}{2} {W'}^3_\mu \delta^b{}_c \\ &\quad + (U_3)^a{}_b i\frac{g_1}{2}\left(\frac{1}{3}\right) {B'}_\mu \delta^b{}_c\Big\} u^c_L \end{aligned} \tag{7}$$

$$ig_3 (U_3)^a{}_b (G_\mu)^b{}_c = [\partial_\mu (U_3)^a{}_c] + ig_3 ({G'}_\mu)^a{}_b (U_3)^b{}_c \tag{8}$$

$$ig_3 ({G'}_\mu)^a{}_b (U_3)^b{}_c = ig_3 (U_3)^a{}_b (G_\mu)^b{}_c - [\partial_\mu (U_3)^a{}_c] \tag{9}$$

$$({G'}_\mu)^a{}_b (U_3)^b{}_c = (U_3)^a{}_b (G_\mu)^b{}_c + \frac{i}{g_3}[\partial_\mu (U_3)^a{}_c] \tag{10}$$

$$({G'}_\mu)^a{}_b (U_3)^b{}_c (U_3^{-1})^c{}_d = (U_3)^a{}_b (G_\mu)^b{}_c (U_3^{-1})^c{}_d + \frac{i}{g_3}[\partial_\mu (U_3)^a{}_c](U_3^{-1})^c{}_d \tag{11}$$

$$({G'}_\mu)^a{}_b \delta^b{}_d = (U_3)^a{}_b (G_\mu)^b{}_c (U_3^{-1})^c{}_d + \frac{i}{g_3}[\partial_\mu (U_3)^a{}_c](U_3^{-1})^c{}_d \tag{12}$$

$$({G'}_\mu)^a{}_d = (U_3)^a{}_b (G_\mu)^b{}_c (U_3^{-1})^c{}_d + \frac{i}{g_3}[\partial_\mu (U_3)^a{}_c](U_3^{-1})^c{}_d \tag{13}$$

$$
(D_\mu)\begin{pmatrix} u_L \\ d_L \end{pmatrix} = \begin{pmatrix} i\gamma^\mu[\partial_\mu + i\frac{g_2}{2}W_\mu^3] & i\gamma^\mu\frac{ig_2}{\sqrt{2}}W_\mu^+ \\ i\gamma^\mu\frac{ig_2}{\sqrt{2}}W_\mu^- & i\gamma^\mu[\partial_\mu - i\frac{g_2}{2}W_\mu^3] \end{pmatrix}\begin{pmatrix} u_L \\ d_L \end{pmatrix} = 0 \tag{14}
$$

$$
\begin{pmatrix} u_L \\ d_L \end{pmatrix} \to \begin{pmatrix} u'_L \\ d'_L \end{pmatrix} = \begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix}\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad U_u{}^u U_d{}^d - U_u{}^d U_d{}^u = 1 \tag{15}
$$

$$
(D'_\mu)\begin{pmatrix} u'_L \\ d'_L \end{pmatrix} = \begin{pmatrix} i\gamma^\mu[\partial_\mu + i\frac{g_2}{2}{W'}^3_\mu] & i\gamma^\mu\frac{ig_2}{\sqrt{2}}{W'}^+_\mu \\ i\gamma^\mu\frac{ig_2}{\sqrt{2}}{W'}^-_\mu & i\gamma^\mu[\partial_\mu - i\frac{g_2}{2}{W'}^3_\mu] \end{pmatrix}\begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix}\begin{pmatrix} u_L \\ d_L \end{pmatrix} \tag{16}
$$

$$
\begin{aligned}
&= i\gamma^\mu\begin{pmatrix} \partial_\mu U_u{}^u & \partial_\mu U_u{}^d \\ \partial_\mu U_d{}^u & \partial_\mu U_d{}^d \end{pmatrix}\begin{pmatrix} u_L \\ d_L \end{pmatrix} \\
&\quad + i\gamma^\mu\begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix}\begin{pmatrix} \partial_\mu u_L \\ \partial_\mu d_L \end{pmatrix} \\
&\quad + \begin{pmatrix} i\gamma^\mu[i\frac{g_2}{2}{W'}^3_\mu] & i\gamma^\mu\frac{ig_2}{\sqrt{2}}{W'}^+_\mu \\ i\gamma^\mu\frac{ig_2}{\sqrt{2}}{W'}^-_\mu & -i\gamma^\mu[i\frac{g_2}{2}{W'}^3_\mu] \end{pmatrix}\begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix}\begin{pmatrix} u_L \\ d_L \end{pmatrix}
\end{aligned} \tag{17}
$$

$$
\begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix}\begin{pmatrix} i\frac{g_2}{2}W_\mu^3 & i\frac{g_2}{\sqrt{2}}W_\mu^+ \\ i\frac{g_2}{\sqrt{2}}W_\mu^- & -i\frac{g_2}{2}W_\mu^3 \end{pmatrix} = \begin{pmatrix} i\frac{g_2}{2}{W'}^3_\mu & i\frac{g_2}{\sqrt{2}}{W'}^+_\mu \\ i\frac{g_2}{\sqrt{2}}{W'}^-_\mu & -i\frac{g_2}{2}{W'}^3_\mu \end{pmatrix}\begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix} + \begin{pmatrix} \partial_\mu U_u{}^u & \partial_\mu U_u{}^d \\ \partial_\mu U_d{}^u & \partial_\mu U_d{}^d \end{pmatrix} \tag{18}
$$

$$
\begin{pmatrix} i\frac{g_2}{2}{W'}^3_\mu & i\frac{g_2}{\sqrt{2}}{W'}^+_\mu \\ i\frac{g_2}{\sqrt{2}}{W'}^-_\mu & -i\frac{g_2}{2}{W'}^3_\mu \end{pmatrix}\begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix} = \begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix}\begin{pmatrix} i\frac{g_2}{2}W_\mu^3 & i\frac{g_2}{\sqrt{2}}W_\mu^+ \\ i\frac{g_2}{\sqrt{2}}W_\mu^- & -i\frac{g_2}{2}W_\mu^3 \end{pmatrix} - \begin{pmatrix} \partial_\mu U_u{}^u & \partial_\mu U_u{}^d \\ \partial_\mu U_d{}^u & \partial_\mu U_d{}^d \end{pmatrix} \tag{19}
$$

$$
\begin{pmatrix} \frac{1}{2}{W'}^3_\mu & \frac{1}{\sqrt{2}}{W'}^+_\mu \\ \frac{1}{\sqrt{2}}{W'}^-_\mu & -\frac{1}{2}{W'}^3_\mu \end{pmatrix}\begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix} = \begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix}\begin{pmatrix} \frac{1}{2}W_\mu^3 & \frac{1}{\sqrt{2}}W_\mu^+ \\ \frac{1}{\sqrt{2}}W_\mu^- & -\frac{1}{2}W_\mu^3 \end{pmatrix} + \frac{i}{g_2}\begin{pmatrix} \partial_\mu U_u{}^u & \partial_\mu U_u{}^d \\ \partial_\mu U_d{}^u & \partial_\mu U_d{}^d \end{pmatrix} \tag{20}
$$

$$
\begin{aligned}
\begin{pmatrix} \frac{1}{2}{W'}^3_\mu & \frac{1}{\sqrt{2}}{W'}^+_\mu \\ \frac{1}{\sqrt{2}}{W'}^-_\mu & -\frac{1}{2}{W'}^3_\mu \end{pmatrix} &= \begin{pmatrix} U_u{}^u & U_u{}^d \\ U_d{}^u & U_d{}^d \end{pmatrix}\begin{pmatrix} \frac{1}{2}W_\mu^3 & \frac{1}{\sqrt{2}}W_\mu^+ \\ \frac{1}{\sqrt{2}}W_\mu^- & -\frac{1}{2}W_\mu^3 \end{pmatrix}\begin{pmatrix} U_d{}^d & -U_u{}^d \\ -U_d{}^u & U_u{}^u \end{pmatrix} \\
&\quad + \frac{i}{g_2}\begin{pmatrix} \partial_\mu U_u{}^u & \partial_\mu U_u{}^d \\ \partial_\mu U_d{}^u & \partial_\mu U_d{}^d \end{pmatrix}\begin{pmatrix} U_d{}^d & -U_u{}^d \\ -U_d{}^u & U_u{}^u \end{pmatrix}
\end{aligned} \tag{21}
$$

$U(1)_Y$ は省略。

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)