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谷脇 貞善

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1　本文

$$S[\psi] = \int d^4x \mathcal{L}(\psi) = \int d^4x \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x) \tag{1}$$

$$S'[\psi] = \int d^4x \bar{\psi}'(x)(i\gamma'^\mu \partial'_\mu - m)\psi'(x) \tag{2}$$

$$= \int d^4x \bar{\psi}(x)[i\Lambda^\mu{}_\alpha \gamma^\alpha (\Lambda^{-1})^\beta{}_\mu \partial_\beta - m]\psi(x) \tag{3}$$

$$= \int d^4x \bar{\psi}(x)[i\delta^\beta{}_\alpha \gamma^\alpha \partial_\beta - m]\psi(x) \tag{4}$$

$$= \int d^4x \bar{\psi}(x)[i\gamma^\beta \partial_\beta - m]\psi(x) = S[\psi] \tag{5}$$

$$S'[\psi] = \int d^4x \psi^\dagger(x)(i\gamma^\mu \partial_\mu - m)^\dagger \bar{\psi}^\dagger(x) \tag{6}$$

$$= \int d^4x \psi(x)[-iI_4(\gamma^\mu)^\dagger \partial_\mu - m](\gamma^0)^\dagger \psi^*(x) \tag{7}$$

$$= \int d^4x \psi(x)[-i\gamma^0\gamma^0(\gamma^\mu)^\dagger\gamma^0\partial_\mu - \gamma^0 m]\psi^*(x) \tag{8}$$

$$= \int d^4x \psi(x)\gamma^0[-i\gamma^\mu \partial_\mu - m]\psi^*(x) \tag{9}$$

$$= -\int d^4x i\gamma^0\gamma^\mu \partial_\mu[\psi(x)\psi^*(x)] + \int d^4x \psi^*(x)\gamma^0[i\gamma^\mu \partial_\mu - m]\psi(x) \tag{10}$$

$$= 0 + \int d^4x \bar{\psi}(x)[i\gamma^\mu \partial_\mu - m]\psi(x) = S[\psi] \tag{11}$$

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)