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$$\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho} \tag{1}$$

$$\mathcal{L}_{F\tilde{F}} = \alpha F_{\mu\nu} \tilde{F}^{\mu\nu} \tag{2}$$

$$= \alpha F_{\mu\nu} \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho} \tag{3}$$

$$= \alpha \epsilon^{\mu\nu\lambda\rho} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\lambda A_\rho - \partial_\rho A_\lambda) \tag{4}$$

$$\begin{aligned} &= \alpha \epsilon^{\mu\nu\lambda\rho} (\partial_\mu A_\nu)(\partial_\lambda A_\rho) - \alpha \epsilon^{\mu\nu\lambda\rho} (\partial_\nu A_\mu)(\partial_\rho A_\lambda) \\ &\quad - \alpha \epsilon^{\mu\nu\lambda\rho} (\partial_\mu A_\nu)(\partial_\lambda A_\rho) + \alpha \epsilon^{\mu\nu\lambda\rho} (\partial_\nu A_\mu)(\partial_\rho A_\lambda) \end{aligned} \tag{5}$$

$$\begin{aligned} &= \alpha \epsilon^{\mu\nu\lambda\rho} (\partial_\mu A_\nu)(\partial_\lambda A_\rho) + \alpha \epsilon^{\nu\mu\rho\lambda} (\partial_\nu A_\mu)(\partial_\rho A_\lambda) \\ &\quad + \alpha \epsilon^{\mu\nu\lambda\rho} (\partial_\mu A_\nu)(\partial_\lambda A_\rho) + \alpha \epsilon^{\nu\mu\rho\lambda} (\partial_\nu A_\mu)(\partial_\rho A_\lambda) \end{aligned} \tag{6}$$

$$= 4\alpha \epsilon^{\mu\nu\lambda\rho} (\partial_\mu A_\nu)(\partial_\lambda A_\rho) \tag{7}$$

$$4\alpha \epsilon^{\mu\nu\lambda\rho} \partial_\mu (A_\nu \partial_\lambda A_\rho) = 4\alpha \epsilon^{\mu\nu\lambda\rho} (\partial_\mu A_\nu)(\partial_\lambda A_\rho) + 4\alpha \epsilon^{\mu\nu\lambda\rho} (A_\nu)(\partial_\mu \partial_\lambda A_\rho) \tag{8}$$

$$= 4\alpha \epsilon^{\mu\nu\lambda\rho} (\partial_\mu A_\nu)(\partial_\lambda A_\rho) \tag{9}$$

$$\mathcal{L}_{F\tilde{F}} = 4\alpha \epsilon^{\mu\nu\lambda\rho} \partial_\mu (A_\nu \partial_\lambda A_\rho) \tag{10}$$

$$\mathcal{L}_{F\tilde{F}} = \alpha\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} \tag{11}$$

$$\begin{aligned}
&= \alpha\epsilon^{0123}F_{01}F_{23} + \alpha\epsilon^{0132}F_{01}F_{32} + \alpha\epsilon^{0213}F_{02}F_{13} + \alpha\epsilon^{0231}F_{02}F_{31} \\
&\quad + \alpha\epsilon^{0312}F_{03}F_{12} + \alpha\epsilon^{0321}F_{03}F_{21} + \alpha\epsilon^{1023}F_{10}F_{23} + \alpha\epsilon^{1032}F_{10}F_{32} \\
&\quad + \alpha\epsilon^{1203}F_{12}F_{03} + \alpha\epsilon^{1230}F_{12}F_{30} + \alpha\epsilon^{1302}F_{13}F_{02} + \alpha\epsilon^{1320}F_{13}F_{20} \\
&\quad + \alpha\epsilon^{2013}F_{20}F_{13} + \alpha\epsilon^{2031}F_{20}F_{31} + \alpha\epsilon^{2103}F_{21}F_{03} + \alpha\epsilon^{2130}F_{21}F_{30} \\
&\quad + \alpha\epsilon^{2301}F_{23}F_{01} + \alpha\epsilon^{2310}F_{23}F_{10} + \alpha\epsilon^{3012}F_{30}F_{12} + \alpha\epsilon^{3021}F_{30}F_{21} \\
&\quad + \alpha\epsilon^{3102}F_{31}F_{02} + \alpha\epsilon^{3120}F_{31}F_{20} + \alpha\epsilon^{3201}F_{32}F_{01} + \alpha\epsilon^{3210}F_{32}F_{10}
\end{aligned} \tag{12}$$

$$\begin{aligned}
&= \alpha(+1)(-E^1)(+B^1) + \alpha(-1)(-E^1)(-B^1) + \alpha(-1)(-E^2)(-B^2) + \alpha(+1)(-E^2)(+B^2) \\
&\quad + \alpha(+1)(-E^3)(+B^3) + \alpha(-1)(-E^3)(-B^3) + \alpha(-1)(+E^1)(+B^1) + \alpha(+1)(+E^1)(-B^1) \\
&\quad + \alpha(+1)(+B^3)(-E^3) + \alpha(-1)(+B^3)(+E^3) + \alpha(-1)(-B^2)(-E^2) + \alpha(+1)(-B^2)(+E^2) \\
&\quad + \alpha(+1)(+E^2)(-B^2) + \alpha(-1)(+E^2)(+B^2) + \alpha(-1)(-B^3)(-E^3) + \alpha(+1)(-B^3)(+E^3) \\
&\quad + \alpha(+1)(+B^1)(-E^1) + \alpha(-1)(+B^1)(+E^1) + \alpha(-1)(+E^3)(+B^3) + \alpha(+1)(+E^3)(-B^3) \\
&\quad + \alpha(+1)(+B^2)(-E^2) + \alpha(-1)(+B^2)(+E^2) + \alpha(-1)(-B^1)(-E^1) + \alpha(+1)(-B^1)(+E^1)
\end{aligned} \tag{13}$$

$$= -8\boldsymbol{E}\cdot\boldsymbol{B} \tag{14}$$

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)