

場の量子論 p291check10.12

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1 本文

次式を証明する。

$$U_{L_z}(\theta) = \exp\{i\theta L_z\} = \exp\{i\theta(xp_y - yp_x)\} \quad (1)$$

$$f_x(\theta) = U_{L_z}(\theta)xU_{L_z}(\theta)^{-1} = \cos\theta x - \sin\theta y \quad (2)$$

$$f_y(\theta) = U_{L_z}(\theta)yU_{L_z}(\theta)^{-1} = \sin\theta x + \cos\theta y \quad (3)$$

$$f_z(\theta) = U_{L_z}(\theta)zU_{L_z}(\theta)^{-1} = z \quad (4)$$

$$f_+(\theta) = U_{L_z}(\theta)(x + iy)U_{L_z}(\theta)^{-1} = e^{i\theta}(x + iy) \quad (5)$$

$$f_-(\theta) = U_{L_z}(\theta)(x - iy)U_{L_z}(\theta)^{-1} = e^{-i\theta}(x - iy) \quad (6)$$

証明。

$$\frac{f_+(\theta)}{d\theta} = \frac{dU_{L_z}(\theta)}{d\theta}(x + iy)U_{L_z}(\theta)^{-1} + U_{L_z}(\theta)(x + iy)\frac{dU_{L_z}(\theta)^{-1}}{d\theta} \quad (7)$$

$$= U_{L_z}(\theta)\{i(xp_y - yp_x)(x + iy) - i(x + iy)(xp_y - yp_x)\}U_{L_z}(\theta)^{-1} \quad (8)$$

$$= U_{L_z}(\theta)\{ixi[p_y, y] - yi[p_x, x]\}U_{L_z}(\theta)^{-1} \quad (9)$$

$$= U_{L_z}(\theta)(ix - y)U_{L_z}(\theta)^{-1} \quad (10)$$

$$= iU_{L_z}(\theta)(x + iy)U_{L_z}(\theta)^{-1} \quad (11)$$

$$= if_+(\theta) \quad (12)$$

$$f_+(\theta) = e^{i\theta}f_+(0) \quad (13)$$

$$= e^{i\theta}(x + iy) \quad (14)$$

$$\frac{f_-(\theta)}{d\theta} = \frac{dU_{L_z}(\theta)}{d\theta}(x - iy)U_{L_z}(\theta)^{-1} + U_{L_z}(\theta)(x - iy)\frac{dU_{L_z}(\theta)^{-1}}{d\theta} \quad (15)$$

$$= U_{L_z}(\theta)\{i(xp_y - yp_x)(x - iy) - i(x - iy)(xp_y - yp_x)\}U_{L_z}(\theta)^{-1} \quad (16)$$

$$= U_{L_z}(\theta)\{-ixi[p_y, y] - yi[p_x, x]\}U_{L_z}(\theta)^{-1} \quad (17)$$

$$= U_{L_z}(\theta)(-ix - y)U_{L_z}(\theta)^{-1} \quad (18)$$

$$= -iU_{L_z}(\theta)(x - iy)U_{L_z}(\theta)^{-1} \quad (19)$$

$$= -if_-(\theta) \quad (20)$$

$$f_-(\theta) = e^{-i\theta}f_-(0) \quad (21)$$

$$= e^{-i\theta}(x - iy) \quad (22)$$

$$f_x(\theta) = \frac{1}{2}[f_+(\theta) + f_-(\theta)] \quad (23)$$

$$= \frac{1}{2}[(e^{i\theta} + e^{-i\theta})x + i(e^{i\theta} - e^{-i\theta})y] \quad (24)$$

$$= \cos \theta x - \sin \theta y \quad (25)$$

$$f_y(\theta) = -\frac{i}{2}[f_+(\theta) - f_-(\theta)] \quad (26)$$

$$= -\frac{i}{2}[(e^{i\theta} - e^{-i\theta})x + i(e^{i\theta} + e^{-i\theta})y] \quad (27)$$

$$= \sin \theta x + \cos \theta y \quad (28)$$

$$f_z(\theta) = U_{L_z}(\theta)zU_{L_z}(\theta)^{-1} \quad (29)$$

$$= zU_{L_z}(\theta)U_{L_z}(\theta)^{-1} \quad (30)$$

$$= z \quad (31)$$

参考文献

- [1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)