# 場の量子論 p311check11.2

谷脇 貞善

2024 年 9 月 26 日

## 1 本文

$$\int_{-\infty}^{+\infty} dx f(x)\delta'(x) = [f(x)\delta(x)]_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} dx f'(x)\delta(x) \tag{1}$$

$$= -f'(0) \tag{2}$$

$$[\phi(\boldsymbol{x}), \pi(\boldsymbol{y})] = i\delta^3(\boldsymbol{x}-\boldsymbol{y}) \tag{3}$$

$$[\phi(\boldsymbol{x}), \phi(\boldsymbol{y})] = 0 = [\pi(\boldsymbol{x}), \pi(\boldsymbol{y})] \tag{4}$$

$$H = \int d^3\boldsymbol{x}\{\frac{1}{2}\pi^2(\boldsymbol{x}) + \frac{1}{2}[\nabla\phi(\boldsymbol{x})]^2 + \frac{m^2}{2}\phi^2(\boldsymbol{x}) + \frac{\lambda}{2}\phi^4(\boldsymbol{x})\} \tag{5}$$

$$= \int d^3\boldsymbol{x}\{\frac{1}{2}\pi^2(\boldsymbol{x}) + \frac{1}{2}\sum_{k=1}^{3}\partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})\partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x}) + V[\phi(\boldsymbol{x})]\} \tag{6}$$

$$P^j = \int d^3\boldsymbol{x}\pi(\boldsymbol{x})\partial_{\boldsymbol{x}}^j\phi(\boldsymbol{x}) \tag{7}$$

$$[P^j, H] = [\int d^3\boldsymbol{y}\pi(\boldsymbol{y})\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), \int d^3\boldsymbol{x}\{\frac{1}{2}\pi^2(\boldsymbol{x}) + \frac{1}{2}\sum_{k=1}^{3}\partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})\partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x}) + V[\phi(\boldsymbol{x})]\}] \tag{8}$$

$$\begin{aligned}&= \int d^3\boldsymbol{x}\int d^3\boldsymbol{y}\{\frac{1}{2}[\pi(\boldsymbol{y})\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), \pi^2(\boldsymbol{x})] + \frac{1}{2}\sum_{k=1}^{3}[\pi(\boldsymbol{y})\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), \partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})\partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})]\\&\quad + [\pi(\boldsymbol{y})\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), V\{\phi(\boldsymbol{x})\}]\}\end{aligned} \tag{9}$$

$$\begin{aligned}&= \int d^3\boldsymbol{x}\int d^3\boldsymbol{y}\{\frac{1}{2}[\pi(\boldsymbol{y})\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), \pi(\boldsymbol{x})]\pi(\boldsymbol{x}) + \frac{1}{2}\pi(\boldsymbol{x})[\pi(\boldsymbol{y})\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), \pi(\boldsymbol{x})]\\&\quad + \frac{1}{2}\sum_{k=1}^{3}\partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})[\pi(\boldsymbol{y})\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), \partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})] + \frac{1}{2}\sum_{k=1}^{3}[\pi(\boldsymbol{y})\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), \partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})]\partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})\\&\quad + [\pi(\boldsymbol{y})\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), \phi(\boldsymbol{x})]\frac{\partial V[\phi(\boldsymbol{x})]}{\partial\phi(\boldsymbol{x})}\}\end{aligned} \tag{10}$$

$$\begin{aligned}&= \int d^3\boldsymbol{x}\int d^3\boldsymbol{y}\{\frac{1}{2}\pi(\boldsymbol{y})[\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), \pi(\boldsymbol{x})]\pi(\boldsymbol{x}) + \frac{1}{2}\pi(\boldsymbol{x})\pi(\boldsymbol{y})[\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}), \pi(\boldsymbol{x})]\\&\quad + \frac{1}{2}\sum_{k=1}^{3}\partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})[\pi(\boldsymbol{y}), \partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})]\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y}) + \frac{1}{2}\sum_{k=1}^{3}[\pi(\boldsymbol{y}), \partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})]\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y})\partial_{\boldsymbol{x}}^k\phi(\boldsymbol{x})\\&\quad + [\pi(\boldsymbol{y}), \phi(\boldsymbol{x})]\partial_{\boldsymbol{y}}^j\phi(\boldsymbol{y})\frac{\partial V[\phi(\boldsymbol{x})]}{\partial\phi(\boldsymbol{x})}\}\end{aligned} \tag{11}$$

$$
\begin{aligned}
[P^j, H] &= \int d^3\boldsymbol{x} \int d^3\boldsymbol{y}\{\frac{1}{2}\pi(\boldsymbol{y})[\partial^j_{\boldsymbol{y}} i\delta(\boldsymbol{y}-\boldsymbol{x})]\pi(\boldsymbol{x}) + \frac{1}{2}\pi(\boldsymbol{x})\pi(\boldsymbol{y})[\partial^j_{\boldsymbol{y}} i\delta(\boldsymbol{y}-\boldsymbol{x})] \\
&\quad + \frac{1}{2}\sum_{k=1}^{3}\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x})[\partial^k_{\boldsymbol{x}}(-i)\delta(\boldsymbol{x}-\boldsymbol{y})]\partial^j_{\boldsymbol{y}}\phi(\boldsymbol{y}) + \frac{1}{2}\sum_{k=1}^{3}[\partial^k_{\boldsymbol{x}}(-i)\delta(\boldsymbol{x}-\boldsymbol{y})]\partial^j_{\boldsymbol{y}}\phi(\boldsymbol{y})\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x}) \\
&\quad + [-i\delta(\boldsymbol{x}-\boldsymbol{y})]\partial^j_{\boldsymbol{y}}\phi(\boldsymbol{y})\frac{\partial V[\phi(\boldsymbol{x})]}{\partial\phi(\boldsymbol{x})}\} && (12) \\
&= \int d^3\boldsymbol{x}\{\frac{1}{2}(-i)\partial^j_{\boldsymbol{x}}[\pi(\boldsymbol{x})\pi(\boldsymbol{x})] + \frac{1}{2}(-i)\partial^j_{\boldsymbol{x}}[\pi(\boldsymbol{x})\pi(\boldsymbol{x})] \\
&\quad + \frac{1}{2}\sum_{k=1}^{3} i\partial^k_{\boldsymbol{x}}[\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x})\partial^j_{\boldsymbol{x}}\phi(\boldsymbol{x})] + \frac{1}{2}\sum_{k=1}^{3} i\partial^k_{\boldsymbol{x}}[\partial^j_{\boldsymbol{x}}\phi(\boldsymbol{x})\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x})] - i\partial^j_{\boldsymbol{x}}\phi(\boldsymbol{x})\frac{\partial V[\phi(\boldsymbol{x})]}{\partial\phi(\boldsymbol{x})}\} && (13) \\
&= \int d^3\boldsymbol{x}\{(-i)\partial^j_{\boldsymbol{x}}[\pi^2(\boldsymbol{x})] + i\sum_{k=1}^{3}\partial^k_{\boldsymbol{x}}[\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x})\partial^j_{\boldsymbol{x}}\phi(\boldsymbol{x})] - i\partial^j_{\boldsymbol{x}}V[\phi(\boldsymbol{x})]\} && (14) \\
&= 0 && (15) \\
[P^j, P^k] &= [\int d^3\boldsymbol{y}\pi(\boldsymbol{y})\partial^j_{\boldsymbol{y}}\phi(\boldsymbol{y}), \int d^3\boldsymbol{x}\pi(\boldsymbol{x})\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x})] && (16) \\
&= \int d^3\boldsymbol{y}\int d^3\boldsymbol{x}[\pi(\boldsymbol{y})\partial^j_{\boldsymbol{y}}\phi(\boldsymbol{y}), \pi(\boldsymbol{x})\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x})] && (17) \\
&= \int d^3\boldsymbol{y}\int d^3\boldsymbol{x}\{[\pi(\boldsymbol{y})\partial^j_{\boldsymbol{y}}\phi(\boldsymbol{y}), \pi(\boldsymbol{x})]\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x}) + \pi(\boldsymbol{x})[\pi(\boldsymbol{y})\partial^j_{\boldsymbol{y}}\phi(\boldsymbol{y}), \partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x})]\} && (18) \\
&= \int d^3\boldsymbol{y}\int d^3\boldsymbol{x}\{\pi(\boldsymbol{y})[\partial^j_{\boldsymbol{y}}\phi(\boldsymbol{y}), \pi(\boldsymbol{x})]\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x}) + \pi(\boldsymbol{x})[\pi(\boldsymbol{y}), \partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x})]\partial^j_{\boldsymbol{y}}\phi(\boldsymbol{y})\} && (19) \\
&= \int d^3\boldsymbol{y}\int d^3\boldsymbol{x}\{\pi(\boldsymbol{y})[\partial^j_{\boldsymbol{y}} i\delta(\boldsymbol{y}-\boldsymbol{x})]\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x}) + \pi(\boldsymbol{x})[\partial^k_{\boldsymbol{x}}(-i)\delta(\boldsymbol{x}-\boldsymbol{y})]\partial^j_{\boldsymbol{y}}\phi(\boldsymbol{y})\} && (20) \\
&= \int d^3\boldsymbol{x}\{(-i)\partial^j_{\boldsymbol{x}}[\pi(\boldsymbol{x})\partial^k_{\boldsymbol{x}}\phi(\boldsymbol{x})] + i\partial^k_{\boldsymbol{x}}[\pi(\boldsymbol{x})\partial^j_{\boldsymbol{x}}\phi(\boldsymbol{x})]\} && (21) \\
&= 0 && (22)
\end{aligned}
$$

# 参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)