# 場の量子論 p312check11.3

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## 1　本文

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\begin{aligned}
J^{ij} = -J^{ji} &= \int d\boldsymbol{x}\pi(t,\boldsymbol{x})(x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x}) && (1)\\
[J^{ij},\phi(t,\boldsymbol{x})] &= [\int d\boldsymbol{y}\pi(t,\boldsymbol{y})(y^i\partial_y^j - y^j\partial_y^i)\phi(t,\boldsymbol{y}),\phi(t,\boldsymbol{x})] && (2)\\
&= \int d\boldsymbol{y}[\pi(t,\boldsymbol{y}),\phi(t,\boldsymbol{x})](y^i\partial_y^j - y^j\partial_y^i)\phi(t,\boldsymbol{y}) && (3)\\
&= -i\int d\boldsymbol{y}\delta^3(\boldsymbol{x}-\boldsymbol{y})(y^i\partial_y^j - y^j\partial_y^i)\phi(t,\boldsymbol{y}) && (4)\\
&= -i(x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x}) && (5)\\
[J^{ij},J^{kl}] &= [\int d\boldsymbol{x}\pi(t,\boldsymbol{x})(x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x}),\int d\boldsymbol{y}\pi(t,\boldsymbol{y})(y^k\partial_y^l - y^l\partial_y^k)\phi(t,\boldsymbol{y})] && (6)\\
&= \int d\boldsymbol{x}\int d\boldsymbol{y}[\pi(t,\boldsymbol{x})(x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x}),\pi(t,\boldsymbol{y})(y^k\partial_y^l - y^l\partial_y^k)\phi(t,\boldsymbol{y})] && (7)\\
&= \int d\boldsymbol{x}\int d\boldsymbol{y}\{[\pi(t,\boldsymbol{x}),\pi(t,\boldsymbol{y})(y^k\partial_y^l - y^l\partial_y^k)\phi(t,\boldsymbol{y})](x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x})\\
&\quad + \pi(t,\boldsymbol{x})[(x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x}),\pi(t,\boldsymbol{y})(y^k\partial_y^l - y^l\partial_y^k)\phi(t,\boldsymbol{y})]\} && (8)\\
&= \int d\boldsymbol{x}\int d\boldsymbol{y}\{\pi(t,\boldsymbol{y})[\pi(t,\boldsymbol{x}),(y^k\partial_y^l - y^l\partial_y^k)\phi(t,\boldsymbol{y})](x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x})\\
&\quad + \pi(t,\boldsymbol{x})[(x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x}),\pi(t,\boldsymbol{y})](y^k\partial_y^l - y^l\partial_y^k)\phi(t,\boldsymbol{y})\} && (9)\\
&= \int d\boldsymbol{x}\int d\boldsymbol{y}\{\pi(t,\boldsymbol{y})[(y^k\partial_y^l - y^l\partial_y^k)(-i)\delta(\boldsymbol{y}-\boldsymbol{x})](x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x})\\
&\quad + \pi(t,\boldsymbol{x})[(x^i\partial_x^j - x^j\partial_x^i)i\delta(\boldsymbol{x}-\boldsymbol{y})](y^k\partial_y^l - y^l\partial_y^k)\phi(t,\boldsymbol{y})\} && (10)\\
&= \int d\boldsymbol{x}\int d\boldsymbol{y}\{i\delta(\boldsymbol{y}-\boldsymbol{x})(y^k\partial_y^l - y^l\partial_y^k)\pi(t,\boldsymbol{y})(x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x})\\
&\quad - i\delta(\boldsymbol{x}-\boldsymbol{y})(x^i\partial_x^j - x^j\partial_x^i)\pi(t,\boldsymbol{x})(y^k\partial_y^l - y^l\partial_y^k)\phi(t,\boldsymbol{y})\} && (11)\\
&= \int d\boldsymbol{x}\{i[(x^k\partial_x^l - x^l\partial_x^k)\pi(t,\boldsymbol{x})][(x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x})]\\
&\quad - i[(x^i\partial_x^j - x^j\partial_x^i)\pi(t,\boldsymbol{x})][(x^k\partial_x^l - x^l\partial_x^k)\phi(t,\boldsymbol{x})]\} && (12)
\end{aligned}
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\begin{aligned}
[J^{ij}, J^{kl}] &= \int d\boldsymbol{x}\{(-i)\pi(t,\boldsymbol{x})[(x^k\partial_x^l - x^l\partial_x^k)(x^i\partial_x^j - x^j\partial_x^i)\phi(t,\boldsymbol{x})] \\
&\quad + i\pi(t,\boldsymbol{x})[(x^i\partial_x^j - x^j\partial_x^i)(x^k\partial_x^l - x^l\partial_x^k)\phi(t,\boldsymbol{x})]\} && (13) \\
&= \int d\boldsymbol{x}\{(-i)\pi(t,\boldsymbol{x})(x^k\delta^{li}\partial_x^j - x^k\delta^{lj}\partial_x^i - x^l\delta^{ki}\partial_x^j + x^l\delta^{kj}\partial_x^i)\phi(t,\boldsymbol{x}) \\
&\quad + i\pi(t,\boldsymbol{x})(x^i\delta^{jk}\partial_x^l - x^i\delta^{jl}\partial_x^k - x^j\delta^{ik}\partial_x^l + x^j\delta^{il}\partial_x^k)\phi(t,\boldsymbol{x})\} && (14) \\
&= -i(-\delta^{il}J^{jk} + \delta^{jl}J^{ik} + \delta^{ik}J^{jl} - \delta^{jk}J^{il}) && (15) \\
[J^{xy}, \phi] &= -i(x\partial_y - y\partial_x)\phi && (16) \\
\begin{pmatrix} \frac{dx}{d\theta} \\ \frac{dy}{d\theta} \end{pmatrix} &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} && (17) \\
\det\begin{pmatrix} -\lambda & -1 \\ 1 & -\lambda \end{pmatrix} &= \lambda^2 + 1 = 0 && (18) \\
\lambda &= \pm i && (19) \\
\lambda &= i \quad \begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r_0 e^{i(\theta+\theta_0)} \\ -ir_0 e^{i(\theta+\theta_0)} \end{pmatrix} && (20) \\
\lambda &= -i \quad \begin{pmatrix} i & -1 \\ 1 & i \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r_0 e^{-i(\theta+\theta_0)} \\ ir_0 e^{-i(\theta+\theta_0)} \end{pmatrix} && (21) \\
\begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} r_0\cos(\theta+\theta_0) \\ -r_0\sin(\theta+\theta_0) \end{pmatrix} && (22)
\end{aligned}
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## 参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)