場の量子論 p321check11.7

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\begin{aligned}
[a(\boldsymbol{k}), a(\boldsymbol{k}')] &= [(f_{\boldsymbol{k}}^{(+)}|\phi), (f_{\boldsymbol{k}'}^{(+)}|\phi)] && (1)\\
&= [\int d^3\boldsymbol{x}(f_{\boldsymbol{k}}^{(+)}(t,\boldsymbol{x}))^* i\pi(t,\boldsymbol{x}) - i(\partial_0 f_{\boldsymbol{k}}^{(+)}(t,\boldsymbol{x}))^*\phi(t,\boldsymbol{x}),\\
&\quad \int d^3\boldsymbol{y}(f_{\boldsymbol{k}'}^{(+)}(t,\boldsymbol{y}))^* i\pi(t,\boldsymbol{y}) - i(\partial_0 f_{\boldsymbol{k}'}^{(+)}(t,\boldsymbol{y}))^*\phi(t,\boldsymbol{y})] && (2)\\
&= \int d^3\boldsymbol{x}\int d^3\boldsymbol{y}\{(f_{\boldsymbol{k}}^{(+)}(t,\boldsymbol{x}))^*(\partial_0 f_{\boldsymbol{k}'}^{(+)}(t,\boldsymbol{y}))^*[\pi(t,\boldsymbol{x}),\phi(t,\boldsymbol{y})]\\
&\quad + (\partial_0 f_{\boldsymbol{k}}^{(+)}(t,\boldsymbol{x}))^*(f_{\boldsymbol{k}'}^{(+)}(t,\boldsymbol{y}))^*[\phi(t,\boldsymbol{x}),\pi(t,\boldsymbol{y})]\} && (3)\\
&= \int d^3\boldsymbol{x}\int d^3\boldsymbol{y}\{(f_{\boldsymbol{k}}^{(+)}(t,\boldsymbol{x}))^*(i\partial_0 f_{\boldsymbol{k}'}^{(+)}(t,\boldsymbol{y}))^*\delta(\boldsymbol{x}-\boldsymbol{y})\\
&\quad - (i\partial_0 f_{\boldsymbol{k}}^{(+)}(t,\boldsymbol{x}))^*(f_{\boldsymbol{k}'}^{(+)}(t,\boldsymbol{y}))^*\delta(\boldsymbol{x}-\boldsymbol{y})\} && (4)\\
&= \int d^3\boldsymbol{x}\{(f_{\boldsymbol{k}}^{(+)}(t,\boldsymbol{x}))^*(i\partial_0 f_{\boldsymbol{k}'}^{(-)}(t,\boldsymbol{x})) - (i\partial_0 f_{\boldsymbol{k}}^{(+)}(t,\boldsymbol{x}))^* f_{\boldsymbol{k}'}^{(-)}(t,\boldsymbol{x})\} && (5)\\
&= (f_{\boldsymbol{k}}^{(+)}|f_{\boldsymbol{k}'}^{(-)}) && (6)\\
&= 0 && (7)
\end{aligned}
$$

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\begin{aligned}
[a^\dagger(\boldsymbol{k}), a^\dagger(\boldsymbol{k}')] &= [-(f_{\boldsymbol{k}}^{(-)}|\phi), -(f_{\boldsymbol{k}'}^{(-)}|\phi)] &(8)\\
&= [\int d^3\boldsymbol{x}(f_{\boldsymbol{k}}^{(-)}(t,\boldsymbol{x}))^* i\pi(t,\boldsymbol{x}) - i(\partial_0 f_{\boldsymbol{k}}^{(-)}(t,\boldsymbol{x}))^*\phi(t,\boldsymbol{x}),\\
&\quad \int d^3\boldsymbol{y}(f_{\boldsymbol{k}'}^{(-)}(t,\boldsymbol{y}))^* i\pi(t,\boldsymbol{y}) - i(\partial_0 f_{\boldsymbol{k}'}^{(-)}(t,\boldsymbol{y}))^*\phi(t,\boldsymbol{y})] &(9)\\
&= \int d^3\boldsymbol{x}\int d^3\boldsymbol{y}\{(f_{\boldsymbol{k}}^{(-)}(t,\boldsymbol{x}))^*(\partial_0 f_{\boldsymbol{k}'}^{(-)}(t,\boldsymbol{y}))^*[\pi(t,\boldsymbol{x}),\phi(t,\boldsymbol{y})]\\
&\quad + (\partial_0 f_{\boldsymbol{k}}^{(-)}(t,\boldsymbol{x}))^*(f_{\boldsymbol{k}'}^{(-)}(t,\boldsymbol{y}))^*[\phi(t,\boldsymbol{x}),\pi(t,\boldsymbol{y})]\} &(10)\\
&= \int d^3\boldsymbol{x}\int d^3\boldsymbol{y}\{(f_{\boldsymbol{k}}^{(-)}(t,\boldsymbol{x}))^*(i\partial_0 f_{\boldsymbol{k}'}^{(-)}(t,\boldsymbol{y}))^*\delta(\boldsymbol{x}-\boldsymbol{y})\\
&\quad - (i\partial_0 f_{\boldsymbol{k}}^{(-)}(t,\boldsymbol{x}))^*(f_{\boldsymbol{k}'}^{(-)}(t,\boldsymbol{y}))^*\delta(\boldsymbol{x}-\boldsymbol{y})\} &(11)\\
&= \int d^3\boldsymbol{x}\{(f_{\boldsymbol{k}}^{(-)}(t,\boldsymbol{x}))^*(i\partial_0 f_{\boldsymbol{k}'}^{(+)}(t,\boldsymbol{x})) - (i\partial_0 f_{\boldsymbol{k}}^{(-)}(t,\boldsymbol{x}))^* f_{\boldsymbol{k}'}^{(+)}(t,\boldsymbol{x})\} &(12)\\
&= (f_{\boldsymbol{k}}^{(-)}|f_{\boldsymbol{k}'}^{(+)}) &(13)\\
&= 0 &(14)
\end{aligned}
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参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)