

場の量子論 p348check11.16

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$$\Phi(t, \mathbf{x}) \tag{1}$$

$$= \int \frac{d^3 \mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{a_+(\mathbf{k})e^{-ik_\mu x^\mu} + a_-^\dagger(\mathbf{k})e^{ik_\mu x^\mu}\} \tag{2}$$

$$= \int \frac{d^3 \mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{a_+(\mathbf{k})e^{-iE_{\mathbf{k}}t - i\mathbf{k} \cdot \mathbf{x}} + a_-^\dagger(\mathbf{k})e^{iE_{\mathbf{k}}t + \mathbf{k} \cdot \mathbf{x}}\} \tag{3}$$

$$\int d\mathbf{x} \Pi^\dagger(t, \mathbf{x}) \Pi(t, \mathbf{x}) \tag{4}$$

$$= \int d\mathbf{x} \partial^0 \Phi^\dagger(t, \mathbf{x}) \partial^0 \Phi(t, \mathbf{x}) \tag{5}$$

$$\begin{aligned} &= \int d^3 \mathbf{x} \int \frac{d^3 \mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{a_+^\dagger(\mathbf{k})(iE_{\mathbf{k}})e^{iE_{\mathbf{k}}t - i\mathbf{k} \cdot \mathbf{x}} + a_-(\mathbf{k})(-iE_{\mathbf{k}})e^{-iE_{\mathbf{k}}t + i\mathbf{k} \cdot \mathbf{x}}\} \\ &\quad \times \int \frac{d^3 \mathbf{k}'}{\sqrt{(2\pi)^3 2E_{\mathbf{k}'}}} \{a_+(\mathbf{k}')(-iE_{\mathbf{k}'})e^{-iE_{\mathbf{k}'}t + i\mathbf{k}' \cdot \mathbf{x}} + a_-^\dagger(\mathbf{k}')(iE_{\mathbf{k}'})e^{iE_{\mathbf{k}'}t - i\mathbf{k}' \cdot \mathbf{x}}\} \end{aligned} \tag{6}$$

$$\begin{aligned} &= \int \frac{d^3 \mathbf{k} d^3 \mathbf{k}'}{(2\pi)^3} \int d^3 \mathbf{x} \frac{\sqrt{E_{\mathbf{k}} E_{\mathbf{k}'}}}{2} \{-a_+^\dagger(\mathbf{k})a_-^\dagger(\mathbf{k}')e^{iE_{\mathbf{k}}t + iE_{\mathbf{k}'}t - i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k}' \cdot \mathbf{x}} \\ &\quad - a_-(\mathbf{k})a_+(\mathbf{k}')e^{-iE_{\mathbf{k}}t - iE_{\mathbf{k}'}t + i\mathbf{k} \cdot \mathbf{x} + i\mathbf{k}' \cdot \mathbf{x}} \\ &\quad + a_+^\dagger(\mathbf{k})a_+(\mathbf{k}')e^{iE_{\mathbf{k}}t - iE_{\mathbf{k}'}t - i\mathbf{k} \cdot \mathbf{x} + i\mathbf{k}' \cdot \mathbf{x}} \\ &\quad + a_-(\mathbf{k})a_-^\dagger(\mathbf{k}')e^{-iE_{\mathbf{k}}t + iE_{\mathbf{k}'}t + i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k}' \cdot \mathbf{x}}\} \end{aligned} \tag{7}$$

$$= \int d^3 \mathbf{k} \frac{E_{\mathbf{k}}}{2} \{-a_+^\dagger(\mathbf{k})a_-^\dagger(-\mathbf{k})e^{2iE_{\mathbf{k}}t} - a_-(\mathbf{k})a_+(-\mathbf{k})e^{-2iE_{\mathbf{k}}t} + a_+^\dagger(\mathbf{k})a_+(\mathbf{k}) + a_-(\mathbf{k})a_-^\dagger(\mathbf{k})\} \tag{8}$$

$$\int d\mathbf{x} \nabla \Phi^\dagger(t, \mathbf{x}) \nabla \Phi(t, \mathbf{x}) \quad (9)$$

$$= \int d^3\mathbf{x} \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{a_+^\dagger(\mathbf{k})(-i\mathbf{k})e^{iE_{\mathbf{k}}t-i\mathbf{k}\cdot\mathbf{x}} + a_-(\mathbf{k})(i\mathbf{k})e^{-iE_{\mathbf{k}}t+i\mathbf{k}\cdot\mathbf{x}}\} \\ \cdot \int \frac{d^3\mathbf{k}'}{\sqrt{(2\pi)^3 2E_{\mathbf{k}'}}} \{a_+(\mathbf{k}')(i\mathbf{k}')e^{-iE_{\mathbf{k}'}t+i\mathbf{k}'\cdot\mathbf{x}} + a_-^\dagger(\mathbf{k}')(-i\mathbf{k}')e^{iE_{\mathbf{k}'}t-i\mathbf{k}'\cdot\mathbf{x}}\} \quad (10)$$

$$= \int \frac{d^3\mathbf{k}d^3\mathbf{k}'}{(2\pi)^3} \int d^3\mathbf{x} \frac{\mathbf{k} \cdot \mathbf{k}'}{2\sqrt{E_{\mathbf{k}}E_{\mathbf{k}'}}} \{-a_+^\dagger(\mathbf{k})a_-^\dagger(\mathbf{k}')e^{iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}} \\ - a_-(\mathbf{k})a_+(\mathbf{k}')e^{-iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}} \\ + a_+^\dagger(\mathbf{k})a_+(\mathbf{k}')e^{iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}} \\ + a_-(\mathbf{k})a_-^\dagger(\mathbf{k}')e^{-iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}}\} \quad (11)$$

$$= \int d^3\mathbf{k} \frac{\mathbf{k}^2}{2E_{\mathbf{k}}} \{a_+^\dagger(\mathbf{k})a_-^\dagger(-\mathbf{k})e^{2iE_{\mathbf{k}}t} + a_-(\mathbf{k})a_+(-\mathbf{k})e^{-2iE_{\mathbf{k}}t} + a_+^\dagger(\mathbf{k})a_+(\mathbf{k}) + a_-(\mathbf{k})a_-^\dagger(\mathbf{k})\} \quad (12)$$

$$\int d\mathbf{x} m^2 \Phi^\dagger(t, \mathbf{x}) \Phi(t, \mathbf{x}) \quad (13)$$

$$= \int d^3\mathbf{x} m^2 \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{a_+^\dagger(\mathbf{k})e^{iE_{\mathbf{k}}t-i\mathbf{k}\cdot\mathbf{x}} + a_-(\mathbf{k})e^{-iE_{\mathbf{k}}t+i\mathbf{k}\cdot\mathbf{x}}\} \\ \times \int \frac{d^3\mathbf{k}'}{\sqrt{(2\pi)^3 2E_{\mathbf{k}'}}} \{a_+(\mathbf{k}')e^{-iE_{\mathbf{k}'}t+i\mathbf{k}'\cdot\mathbf{x}} + a_-^\dagger(\mathbf{k}')e^{iE_{\mathbf{k}'}t-i\mathbf{k}'\cdot\mathbf{x}}\} \quad (14)$$

$$= m^2 \int \frac{d^3\mathbf{k}d^3\mathbf{k}'}{(2\pi)^3} \int d^3\mathbf{x} \frac{1}{2\sqrt{E_{\mathbf{k}}E_{\mathbf{k}'}}} \{a_+^\dagger(\mathbf{k})a_-^\dagger(\mathbf{k}')e^{iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}} \\ + a_-(\mathbf{k})a_+(\mathbf{k}')e^{-iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}} \\ + a_+^\dagger(\mathbf{k})a_+(\mathbf{k}')e^{iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}} \\ + a_-(\mathbf{k})a_-^\dagger(\mathbf{k}')e^{-iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}}\} \quad (15)$$

$$= \int d^3\mathbf{k} \frac{m^2}{2E_{\mathbf{k}}} \{a_+^\dagger(\mathbf{k})a_-^\dagger(-\mathbf{k})e^{2iE_{\mathbf{k}}t} + a_-(\mathbf{k})a_+(-\mathbf{k})e^{-2iE_{\mathbf{k}}t} + a_+^\dagger(\mathbf{k})a_+(\mathbf{k}) + a_-(\mathbf{k})a_-^\dagger(\mathbf{k})\} \quad (16)$$

$$H = \int d^3\mathbf{x} \{\Pi^\dagger \Pi + \nabla \Phi^\dagger \nabla \Phi + m^2 \Phi^\dagger \Phi\} \quad (17)$$

$$= \int d^3\mathbf{k} E_{\mathbf{k}} \{a_+^\dagger(\mathbf{k})a_+(\mathbf{k}) + a_-(\mathbf{k})a_-^\dagger(\mathbf{k})\} \quad (18)$$

$$- \int d\mathbf{x} \frac{1}{2} \Pi(t, \mathbf{x}) (\nabla \Phi(t, \mathbf{x})) \quad (19)$$

$$= - \int d^3\mathbf{x} \frac{1}{2} \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{ a_+^\dagger(\mathbf{k}) (iE_{\mathbf{k}}) e^{iE_{\mathbf{k}}t - i\mathbf{k} \cdot \mathbf{x}} + a_-(\mathbf{k}) (-iE_{\mathbf{k}}) e^{-iE_{\mathbf{k}}t + i\mathbf{k} \cdot \mathbf{x}} \} \\ \times \int \frac{d^3\mathbf{k}'}{\sqrt{(2\pi)^3 2E_{\mathbf{k}'}}} \{ a_+(\mathbf{k}') (i\mathbf{k}') e^{-iE_{\mathbf{k}'}t + i\mathbf{k}' \cdot \mathbf{x}} + a_-^\dagger(\mathbf{k}') (-i\mathbf{k}') e^{iE_{\mathbf{k}'}t - i\mathbf{k}' \cdot \mathbf{x}} \} \quad (20)$$

$$= \frac{1}{2} \int \frac{d^3\mathbf{k} d^3\mathbf{k}'}{(2\pi)^3} \int d^3\mathbf{x} \frac{E_{\mathbf{k}} \mathbf{k}'}{2\sqrt{E_{\mathbf{k}} E_{\mathbf{k}'}}} \{ -a_+^\dagger(\mathbf{k}) a_-^\dagger(\mathbf{k}') e^{iE_{\mathbf{k}}t + iE_{\mathbf{k}'}t - i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k}' \cdot \mathbf{x}} \\ - a_-(\mathbf{k}) a_+(\mathbf{k}') e^{-iE_{\mathbf{k}}t - iE_{\mathbf{k}'}t + i\mathbf{k} \cdot \mathbf{x} + i\mathbf{k}' \cdot \mathbf{x}} \\ + a_+^\dagger(\mathbf{k}) a_+(\mathbf{k}') e^{iE_{\mathbf{k}}t - iE_{\mathbf{k}'}t - i\mathbf{k} \cdot \mathbf{x} + i\mathbf{k}' \cdot \mathbf{x}} \\ + a_-(\mathbf{k}) a_-^\dagger(\mathbf{k}') e^{-iE_{\mathbf{k}}t + iE_{\mathbf{k}'}t + i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k}' \cdot \mathbf{x}} \} \quad (21)$$

$$= \int d^3\mathbf{k} \frac{\mathbf{k}}{4} \{ a_+^\dagger(\mathbf{k}) a_-^\dagger(-\mathbf{k}) e^{2iE_{\mathbf{k}}t} + a_-(\mathbf{k}) a_+(-\mathbf{k}) e^{-2iE_{\mathbf{k}}t} + a_+^\dagger(\mathbf{k}) a_+(\mathbf{k}) + a_-(\mathbf{k}) a_-^\dagger(\mathbf{k}) \} \quad (22)$$

$$- \int d\mathbf{x} \frac{1}{2} (\nabla \Phi(t, \mathbf{x})) \Pi(t, \mathbf{x}) \quad (23)$$

$$= - \int d^3\mathbf{x} \frac{1}{2} \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{ a_+(\mathbf{k}) (i\mathbf{k}) e^{-iE_{\mathbf{k}}t + i\mathbf{k} \cdot \mathbf{x}} + a_-^\dagger(\mathbf{k}) (-i\mathbf{k}) e^{iE_{\mathbf{k}}t - i\mathbf{k} \cdot \mathbf{x}} \} \\ \cdot \int \frac{d^3\mathbf{k}'}{\sqrt{(2\pi)^3 2E_{\mathbf{k}'}}} \{ a_+^\dagger(\mathbf{k}') (iE_{\mathbf{k}'}) e^{iE_{\mathbf{k}'}t - i\mathbf{k}' \cdot \mathbf{x}} + a_-(\mathbf{k}') (-iE_{\mathbf{k}'}) e^{-iE_{\mathbf{k}'}t + i\mathbf{k}' \cdot \mathbf{x}} \} \quad (24)$$

$$= \frac{1}{2} \int \frac{d^3\mathbf{k} d^3\mathbf{k}'}{(2\pi)^3} \int d^3\mathbf{x} \frac{E_{\mathbf{k}} \mathbf{k}}{2\sqrt{E_{\mathbf{k}} E_{\mathbf{k}'}}} \{ -a_+(\mathbf{k}) a_-(\mathbf{k}') e^{-iE_{\mathbf{k}}t - iE_{\mathbf{k}'}t + i\mathbf{k} \cdot \mathbf{x} + i\mathbf{k}' \cdot \mathbf{x}} \\ - a_-^\dagger(\mathbf{k}) a_+^\dagger(\mathbf{k}') e^{iE_{\mathbf{k}}t + iE_{\mathbf{k}'}t - i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k}' \cdot \mathbf{x}} \\ + a_+(\mathbf{k}) a_+^\dagger(\mathbf{k}') e^{-iE_{\mathbf{k}}t + iE_{\mathbf{k}'}t + i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k}' \cdot \mathbf{x}} \\ + a_-^\dagger(\mathbf{k}) a_-(\mathbf{k}') e^{iE_{\mathbf{k}}t - iE_{\mathbf{k}'}t - i\mathbf{k} \cdot \mathbf{x} + i\mathbf{k}' \cdot \mathbf{x}} \} \quad (25)$$

$$= \int d^3\mathbf{k} \frac{\mathbf{k}}{4} \{ -a_+(\mathbf{k}) a_-(\mathbf{k}) e^{-2iE_{\mathbf{k}}t} - a_-^\dagger(\mathbf{k}) a_+^\dagger(-\mathbf{k}) e^{2iE_{\mathbf{k}}t} + a_+(\mathbf{k}) a_+^\dagger(\mathbf{k}) + a_-^\dagger(\mathbf{k}) a_-(\mathbf{k}) \} \quad (26)$$

$$- \int d\mathbf{x} \frac{1}{2} (\nabla \Phi^\dagger(t, \mathbf{x})) \Pi^\dagger(t, \mathbf{x}) \quad (27)$$

$$= - \int d^3\mathbf{x} \frac{1}{2} \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{a_+^\dagger(\mathbf{k})(-i\mathbf{k})e^{iE_{\mathbf{k}}t-i\mathbf{k}\cdot\mathbf{x}} + a_-(\mathbf{k})(i\mathbf{k})e^{-iE_{\mathbf{k}}t+i\mathbf{k}\cdot\mathbf{x}}\} \\ \cdot \int \frac{d^3\mathbf{k}'}{\sqrt{(2\pi)^3 2E_{\mathbf{k}'}}} \{a_+(\mathbf{k}')(-iE_{\mathbf{k}'})e^{-iE_{\mathbf{k}'}t+i\mathbf{k}'\cdot\mathbf{x}} + a_-^\dagger(\mathbf{k}')(iE_{\mathbf{k}'})e^{iE_{\mathbf{k}'}t-i\mathbf{k}'\cdot\mathbf{x}}\} \quad (28)$$

$$= \frac{1}{2} \int \frac{d^3\mathbf{k}d^3\mathbf{k}'}{(2\pi)^3} \int d^3\mathbf{x} \frac{E_{\mathbf{k}}\mathbf{k}}{2\sqrt{E_{\mathbf{k}}E_{\mathbf{k}'}}} \{-a_+^\dagger(\mathbf{k})a_-^\dagger(\mathbf{k}')e^{iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}} \\ - a_-(\mathbf{k})a_+(\mathbf{k}')e^{-iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}} \\ + a_+^\dagger(\mathbf{k})a_+(\mathbf{k}')e^{iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}} \\ + a_-(\mathbf{k})a_-^\dagger(\mathbf{k}')e^{-iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}}\} \quad (29)$$

$$= \int d^3\mathbf{k} \frac{\mathbf{k}}{4} \{-a_+^\dagger(\mathbf{k})a_-^\dagger(-\mathbf{k})e^{2iE_{\mathbf{k}}t} - a_-(\mathbf{k})a_+(-\mathbf{k})e^{-2iE_{\mathbf{k}}t} + a_+^\dagger(\mathbf{k})a_+(\mathbf{k}) + a_-(\mathbf{k})a_-^\dagger(\mathbf{k})\} \quad (30)$$

$$- \int d\mathbf{x} \frac{1}{2} \Pi^\dagger(t, \mathbf{x}) (\nabla \Phi^\dagger(t, \mathbf{x})) \quad (31)$$

$$= - \int d^3\mathbf{x} \frac{1}{2} \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{a_+(\mathbf{k})(-iE_{\mathbf{k}})e^{-iE_{\mathbf{k}}t+i\mathbf{k}\cdot\mathbf{x}} + a_-^\dagger(\mathbf{k})(iE_{\mathbf{k}})e^{iE_{\mathbf{k}}t-i\mathbf{k}\cdot\mathbf{x}}\} \\ \times \int \frac{d^3\mathbf{k}'}{\sqrt{(2\pi)^3 2E_{\mathbf{k}'}}} \{a_+^\dagger(\mathbf{k}')(-i\mathbf{k}')e^{iE_{\mathbf{k}'}t-i\mathbf{k}'\cdot\mathbf{x}} + a_-(\mathbf{k}')(i\mathbf{k}')e^{-iE_{\mathbf{k}'}t+i\mathbf{k}'\cdot\mathbf{x}}\} \quad (32)$$

$$= \frac{1}{2} \int \frac{d^3\mathbf{k}d^3\mathbf{k}'}{(2\pi)^3} \int d^3\mathbf{x} \frac{E_{\mathbf{k}}\mathbf{k}'}{2\sqrt{E_{\mathbf{k}}E_{\mathbf{k}'}}} \{-a_+(\mathbf{k})a_-(\mathbf{k}')e^{-iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}} \\ - a_-^\dagger(\mathbf{k})a_+^\dagger(\mathbf{k}')e^{iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}} \\ + a_+(\mathbf{k})a_+^\dagger(\mathbf{k}')e^{-iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}} \\ + a_-^\dagger(\mathbf{k})a_-(\mathbf{k}')e^{iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}}\} \quad (33)$$

$$= \int d^3\mathbf{k} \frac{\mathbf{k}}{4} \{a_+(\mathbf{k})a_-(\mathbf{k})e^{-2iE_{\mathbf{k}}t} + a_-^\dagger(\mathbf{k})a_+^\dagger(-\mathbf{k})e^{2iE_{\mathbf{k}}t} + a_+(\mathbf{k})a_+^\dagger(\mathbf{k}) + a_-^\dagger(\mathbf{k})a_-(\mathbf{k})\} \quad (34)$$

$$P = -\frac{1}{2} \int d^3\mathbf{x} \{\Pi(\nabla \Phi) + (\nabla \Phi)\Pi + (\nabla \Phi)^\dagger \Pi^\dagger + \Pi^\dagger(\nabla \Phi)^\dagger\} \quad (35)$$

$$= \int d^3\mathbf{k} \mathbf{k} \{a_+^\dagger(\mathbf{k})a_+(\mathbf{k}) + a_-^\dagger(\mathbf{k})a_-(\mathbf{k})\} \quad (36)$$

$$iq \int d\mathbf{x} \frac{1}{2} \Phi^\dagger(t, \mathbf{x}) \Pi^\dagger(t, \mathbf{x}) \quad (37)$$

$$= iq \int d^3\mathbf{x} \frac{1}{2} \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3} 2E_{\mathbf{k}}} \{a_+^\dagger(\mathbf{k}) e^{iE_{\mathbf{k}}t - i\mathbf{k} \cdot \mathbf{x}} + a_-(\mathbf{k}) e^{-iE_{\mathbf{k}}t + i\mathbf{k} \cdot \mathbf{x}}\} \\ \cdot \int \frac{d^3\mathbf{k}'}{\sqrt{(2\pi)^3} 2E_{\mathbf{k}'}} \{a_+(\mathbf{k}') (-iE_{\mathbf{k}'}) e^{-iE_{\mathbf{k}'}t + i\mathbf{k}' \cdot \mathbf{x}} + a_-^\dagger(\mathbf{k}') (iE_{\mathbf{k}'}) e^{iE_{\mathbf{k}'}t - i\mathbf{k}' \cdot \mathbf{x}}\} \quad (38)$$

$$= \frac{q}{2} \int \frac{d^3\mathbf{k} d^3\mathbf{k}'}{(2\pi)^3} \int d^3\mathbf{x} \frac{E_{\mathbf{k}'}}{2\sqrt{E_{\mathbf{k}} E_{\mathbf{k}'}}} \{-a_+^\dagger(\mathbf{k}) a_-^\dagger(\mathbf{k}') e^{iE_{\mathbf{k}}t + iE_{\mathbf{k}'}t - i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k}' \cdot \mathbf{x}} \\ + a_-(\mathbf{k}) a_+(\mathbf{k}') e^{-iE_{\mathbf{k}}t - iE_{\mathbf{k}'}t + i\mathbf{k} \cdot \mathbf{x} + i\mathbf{k}' \cdot \mathbf{x}} \\ + a_+^\dagger(\mathbf{k}) a_+(\mathbf{k}') e^{iE_{\mathbf{k}}t - iE_{\mathbf{k}'}t - i\mathbf{k} \cdot \mathbf{x} + i\mathbf{k}' \cdot \mathbf{x}} \\ - a_-(\mathbf{k}) a_-^\dagger(\mathbf{k}') e^{-iE_{\mathbf{k}}t + iE_{\mathbf{k}'}t + i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k}' \cdot \mathbf{x}}\} \quad (39)$$

$$= \int d^3\mathbf{k} \frac{q}{4} \{-a_+^\dagger(\mathbf{k}) a_-^\dagger(-\mathbf{k}) e^{2iE_{\mathbf{k}}t} + a_-(\mathbf{k}) a_+(-\mathbf{k}) e^{-2iE_{\mathbf{k}}t} + a_+^\dagger(\mathbf{k}) a_+(\mathbf{k}) - a_-(\mathbf{k}) a_-^\dagger(\mathbf{k})\} \quad (40)$$

$$iq \int d\mathbf{x} \frac{1}{2} \Pi^\dagger(t, \mathbf{x}) \Phi^\dagger(t, \mathbf{x}) \quad (41)$$

$$= iq \int d^3\mathbf{x} \frac{1}{2} \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3} 2E_{\mathbf{k}}} \{a_+(\mathbf{k}) (-iE_{\mathbf{k}}) e^{-iE_{\mathbf{k}}t + i\mathbf{k} \cdot \mathbf{x}} + a_-^\dagger(\mathbf{k}) (iE_{\mathbf{k}}) e^{iE_{\mathbf{k}}t - i\mathbf{k} \cdot \mathbf{x}}\} \\ \cdot \int \frac{d^3\mathbf{k}'}{\sqrt{(2\pi)^3} 2E_{\mathbf{k}'}} \{a_+^\dagger(\mathbf{k}') e^{iE_{\mathbf{k}'}t - i\mathbf{k}' \cdot \mathbf{x}} + a_-(\mathbf{k}') e^{-iE_{\mathbf{k}'}t + i\mathbf{k}' \cdot \mathbf{x}}\} \quad (42)$$

$$= \frac{q}{2} \int \frac{d^3\mathbf{k} d^3\mathbf{k}'}{(2\pi)^3} \int d^3\mathbf{x} \frac{E_{\mathbf{k}}}{2\sqrt{E_{\mathbf{k}} E_{\mathbf{k}'}}} \{-a_-^\dagger(\mathbf{k}) a_+^\dagger(\mathbf{k}') e^{iE_{\mathbf{k}}t + iE_{\mathbf{k}'}t - i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k}' \cdot \mathbf{x}} \\ + a_+(\mathbf{k}) a_-(\mathbf{k}') e^{-iE_{\mathbf{k}}t - iE_{\mathbf{k}'}t + i\mathbf{k} \cdot \mathbf{x} + i\mathbf{k}' \cdot \mathbf{x}} \\ - a_-^\dagger(\mathbf{k}) a_-(\mathbf{k}') e^{iE_{\mathbf{k}}t - iE_{\mathbf{k}'}t - i\mathbf{k} \cdot \mathbf{x} + i\mathbf{k}' \cdot \mathbf{x}} \\ + a_+(\mathbf{k}) a_+^\dagger(\mathbf{k}') e^{-iE_{\mathbf{k}}t + iE_{\mathbf{k}'}t + i\mathbf{k} \cdot \mathbf{x} - i\mathbf{k}' \cdot \mathbf{x}}\} \quad (43)$$

$$= \int d^3\mathbf{k} \frac{q}{4} \{-a_-^\dagger(\mathbf{k}) a_+^\dagger(-\mathbf{k}) e^{2iE_{\mathbf{k}}t} + a_+(\mathbf{k}) a_-(-\mathbf{k}) e^{-2iE_{\mathbf{k}}t} - a_-^\dagger(\mathbf{k}) a_-(\mathbf{k}) + a_+(\mathbf{k}) a_+^\dagger(\mathbf{k})\} \quad (44)$$

$$-iq \int d\mathbf{x} \frac{1}{2} \Pi(t, \mathbf{x}) \Phi(t, \mathbf{x}) \quad (45)$$

$$= -iq \int d^3\mathbf{x} \frac{1}{2} \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{a_+^\dagger(\mathbf{k})(iE_{\mathbf{k}})e^{iE_{\mathbf{k}}t-i\mathbf{k}\cdot\mathbf{x}} + a_-(\mathbf{k})(-iE_{\mathbf{k}})e^{-iE_{\mathbf{k}}t+i\mathbf{k}\cdot\mathbf{x}}\} \\ \cdot \int \frac{d^3\mathbf{k}'}{\sqrt{(2\pi)^3 2E_{\mathbf{k}'}}} \{a_+(\mathbf{k}')e^{-iE_{\mathbf{k}'}t+i\mathbf{k}'\cdot\mathbf{x}} + a_-^\dagger(\mathbf{k}')e^{iE_{\mathbf{k}'}t-i\mathbf{k}'\cdot\mathbf{x}}\} \quad (46)$$

$$= \frac{q}{2} \int \frac{d^3\mathbf{k}d^3\mathbf{k}'}{(2\pi)^3} \int d^3\mathbf{x} \frac{E_{\mathbf{k}}}{2\sqrt{E_{\mathbf{k}}E_{\mathbf{k}'}}} \{-a_-(\mathbf{k})a_+(\mathbf{k}')e^{-iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}} \\ + a_+^\dagger(\mathbf{k})a_-^\dagger(\mathbf{k}')e^{iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}} \\ - a_-(\mathbf{k})a_-^\dagger(\mathbf{k}')e^{-iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}} \\ + a_+^\dagger(\mathbf{k})a_+(\mathbf{k}')e^{iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}}\} \quad (47)$$

$$= \int d^3\mathbf{k} \frac{q}{4} \{-a_-(\mathbf{k})a_+(-\mathbf{k})e^{-2iE_{\mathbf{k}}t} + a_+^\dagger(\mathbf{k})a_-^\dagger(-\mathbf{k})e^{2iE_{\mathbf{k}}t} - a_-(\mathbf{k})a_-^\dagger(\mathbf{k}) + a_+^\dagger(\mathbf{k})a_+(\mathbf{k})\} \quad (48)$$

$$-iq \int d\mathbf{x} \frac{1}{2} \Phi(t, \mathbf{x}) \Pi(t, \mathbf{x}) \quad (49)$$

$$= -iq \int d^3\mathbf{x} \frac{1}{2} \int \frac{d^3\mathbf{k}}{\sqrt{(2\pi)^3 2E_{\mathbf{k}}}} \{a_+(\mathbf{k})e^{-iE_{\mathbf{k}}t+i\mathbf{k}\cdot\mathbf{x}} + a_-^\dagger(\mathbf{k})e^{iE_{\mathbf{k}}t-i\mathbf{k}\cdot\mathbf{x}}\} \\ \cdot \int \frac{d^3\mathbf{k}'}{\sqrt{(2\pi)^3 2E_{\mathbf{k}'}}} \{a_+^\dagger(\mathbf{k}')(iE_{\mathbf{k}'})e^{iE_{\mathbf{k}'}t-i\mathbf{k}'\cdot\mathbf{x}} + a_-(\mathbf{k}')(-iE_{\mathbf{k}'})e^{-iE_{\mathbf{k}'}t+i\mathbf{k}'\cdot\mathbf{x}}\} \quad (50)$$

$$= \frac{q}{2} \int \frac{d^3\mathbf{k}d^3\mathbf{k}'}{(2\pi)^3} \int d^3\mathbf{x} \frac{E_{\mathbf{k}'}}{2\sqrt{E_{\mathbf{k}}E_{\mathbf{k}'}}} \{-a_+(\mathbf{k})a_-(\mathbf{k}')e^{-iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}} \\ + a_-^\dagger(\mathbf{k})a_+^\dagger(\mathbf{k}')e^{iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}} \\ + a_+(\mathbf{k})a_+^\dagger(\mathbf{k}')e^{-iE_{\mathbf{k}}t+iE_{\mathbf{k}'}t+i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}} \\ - a_-^\dagger(\mathbf{k})a_-(\mathbf{k}')e^{iE_{\mathbf{k}}t-iE_{\mathbf{k}'}t-i\mathbf{k}\cdot\mathbf{x}+i\mathbf{k}'\cdot\mathbf{x}}\} \quad (51)$$

$$= \int d^3\mathbf{k} \frac{q}{4} \{-a_+(\mathbf{k})a_-(-\mathbf{k})e^{-2iE_{\mathbf{k}}t} + a_-^\dagger(\mathbf{k})a_+^\dagger(-\mathbf{k})e^{2iE_{\mathbf{k}}t} + a_+(\mathbf{k})a_+^\dagger(\mathbf{k}) - a_-^\dagger(\mathbf{k})a_-(\mathbf{k})\} \quad (52)$$

$$Q = iq \int d^3\mathbf{x} \frac{1}{2} \{\Phi^\dagger \Pi^\dagger + \Pi^\dagger \Phi^\dagger - \Pi \Phi - \Phi \Pi\} \quad (53)$$

$$= \int d^3\mathbf{k} q \{a_+^\dagger(\mathbf{k})a_+(\mathbf{k}) - a_-^\dagger(\mathbf{k})a_-(\mathbf{k})\} \quad (54)$$

$$H = \int d^3\mathbf{k}' E_{\mathbf{k}'} \{a_+^\dagger(\mathbf{k}')a_+(\mathbf{k}') + a_-^\dagger(\mathbf{k}')a_-(\mathbf{k}')\} \quad (55)$$

$$P = \int d^3\mathbf{k}' \mathbf{k}' \{a_+^\dagger(\mathbf{k}')a_+(\mathbf{k}') + a_-^\dagger(\mathbf{k}')a_-(\mathbf{k}')\} \quad (56)$$

$$Q = \int d^3\mathbf{k}' q \{a_+^\dagger(\mathbf{k}')a_+(\mathbf{k}') - a_-^\dagger(\mathbf{k}')a_-(\mathbf{k}')\} \quad (57)$$

$$H|0\rangle = P|0\rangle = Q|0\rangle = 0 \quad (58)$$

$$[H, a_+^\dagger(\mathbf{k})] = \int d^3\mathbf{k}' E_{\mathbf{k}'} a_+^\dagger(\mathbf{k}') [a_+(\mathbf{k}'), a_+^\dagger(\mathbf{k})] \quad (59)$$

$$= \int d^3\mathbf{k}' E_{\mathbf{k}'} a_+^\dagger(\mathbf{k}') \delta(\mathbf{k}' - \mathbf{k}) \quad (60)$$

$$= E_{\mathbf{k}} a_+^\dagger(\mathbf{k}) \quad (61)$$

$$[H, a_-^\dagger(\mathbf{k})] = \int d^3\mathbf{k}' E_{\mathbf{k}'} a_-^\dagger(\mathbf{k}') [a_-(\mathbf{k}'), a_-^\dagger(\mathbf{k})] \quad (62)$$

$$= \int d^3\mathbf{k}' E_{\mathbf{k}'} a_-^\dagger(\mathbf{k}') \delta(\mathbf{k}' - \mathbf{k}) \quad (63)$$

$$= E_{\mathbf{k}} a_-^\dagger(\mathbf{k}) \quad (64)$$

$$[P, a_+^\dagger(\mathbf{k})] = \int d^3\mathbf{k}' \mathbf{k}' a_+^\dagger(\mathbf{k}') [a_+(\mathbf{k}'), a_+^\dagger(\mathbf{k})] \quad (65)$$

$$= \int d^3\mathbf{k}' \mathbf{k}' a_+^\dagger(\mathbf{k}') \delta(\mathbf{k}' - \mathbf{k}) \quad (66)$$

$$= \mathbf{k} a_+^\dagger(\mathbf{k}) \quad (67)$$

$$[P, a_-^\dagger(\mathbf{k})] = \int d^3\mathbf{k}' \mathbf{k}' a_-^\dagger(\mathbf{k}') [a_-(\mathbf{k}'), a_-^\dagger(\mathbf{k})] \quad (68)$$

$$= \int d^3\mathbf{k}' \mathbf{k}' a_-^\dagger(\mathbf{k}') \delta(\mathbf{k}' - \mathbf{k}) \quad (69)$$

$$= \mathbf{k} a_-^\dagger(\mathbf{k}) \quad (70)$$

$$[Q, a_+^\dagger(\mathbf{k})] = \int d^3\mathbf{k}' q a_+^\dagger(\mathbf{k}') [a_+(\mathbf{k}'), a_+^\dagger(\mathbf{k})] \quad (71)$$

$$= \int d^3\mathbf{k}' q a_+^\dagger(\mathbf{k}') \delta(\mathbf{k}' - \mathbf{k}) \quad (72)$$

$$= q a_+^\dagger(\mathbf{k}) \quad (73)$$

$$[Q, a_-^\dagger(\mathbf{k})] = - \int d^3\mathbf{k}' q a_-^\dagger(\mathbf{k}') [a_-(\mathbf{k}'), a_-^\dagger(\mathbf{k})] \quad (74)$$

$$= - \int d^3\mathbf{k}' q a_-^\dagger(\mathbf{k}') \delta(\mathbf{k}' - \mathbf{k}) \quad (75)$$

$$= -q a_-^\dagger(\mathbf{k}) \quad (76)$$

$$H|\mathbf{k}, +q\rangle = Ha_+^\dagger(\mathbf{k})|0\rangle - a_+^\dagger(\mathbf{k})H|0\rangle \quad (77)$$

$$= [H, a_+^\dagger(\mathbf{k})]|0\rangle \quad (78)$$

$$= E_{\mathbf{k}}a_+^\dagger(\mathbf{k})|0\rangle \quad (79)$$

$$= E_{\mathbf{k}}|\mathbf{k}, +q\rangle \quad (80)$$

$$H|\mathbf{k}, -q\rangle = Ha_-^\dagger(\mathbf{k})|0\rangle - a_-^\dagger(\mathbf{k})H|0\rangle \quad (81)$$

$$= [H, a_-^\dagger(\mathbf{k})]|0\rangle \quad (82)$$

$$= E_{\mathbf{k}}a_-^\dagger(\mathbf{k})|0\rangle \quad (83)$$

$$= E_{\mathbf{k}}|\mathbf{k}, -q\rangle \quad (84)$$

$$P|\mathbf{k}, +q\rangle = Pa_+^\dagger(\mathbf{k})|0\rangle - a_+^\dagger(\mathbf{k})P|0\rangle \quad (85)$$

$$= [P, a_+^\dagger(\mathbf{k})]|0\rangle \quad (86)$$

$$= \mathbf{k}a_+^\dagger(\mathbf{k})|0\rangle \quad (87)$$

$$= \mathbf{k}|\mathbf{k}, +q\rangle \quad (88)$$

$$P|\mathbf{k}, -q\rangle = Pa_-^\dagger(\mathbf{k})|0\rangle - a_-^\dagger(\mathbf{k})P|0\rangle \quad (89)$$

$$= [P, a_-^\dagger(\mathbf{k})]|0\rangle \quad (90)$$

$$= \mathbf{k}a_-^\dagger(\mathbf{k})|0\rangle \quad (91)$$

$$= \mathbf{k}|\mathbf{k}, -q\rangle \quad (92)$$

$$Q|\mathbf{k}, +q\rangle = Qa_+^\dagger(\mathbf{k})|0\rangle - a_+^\dagger(\mathbf{k})Q|0\rangle \quad (93)$$

$$= [Q, a_+^\dagger(\mathbf{k})]|0\rangle \quad (94)$$

$$= +qa_+^\dagger(\mathbf{k})|0\rangle \quad (95)$$

$$= +q|\mathbf{k}, +q\rangle \quad (96)$$

$$Q|\mathbf{k}, -q\rangle = Qa_-^\dagger(\mathbf{k})|0\rangle - a_-^\dagger(\mathbf{k})Q|0\rangle \quad (97)$$

$$= [Q, a_-^\dagger(\mathbf{k})]|0\rangle \quad (98)$$

$$= -qa_-^\dagger(\mathbf{k})|0\rangle \quad (99)$$

$$= -q|\mathbf{k}, -q\rangle \quad (100)$$

参考文献

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