場の量子論 p356check12.2

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1　本文

$$\sigma^1\sigma^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = i\sigma_3 \tag{1}$$

$$C = i\gamma^2\gamma^0 = i\begin{pmatrix} 0 & -\sigma_2 \\ \sigma_2 & 0 \end{pmatrix}\begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix} = i\begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix} \tag{2}$$

$$= -C^{-1} = -C^\dagger = -C^T = C^* \tag{3}$$

$$\phi^C = C\bar{\phi}^T \tag{4}$$

$$\bar{\phi}^T = C^{-1}\phi^C \tag{5}$$

$$\phi^T = \bar{\phi}^C C^{-1} = -\bar{\phi^C}C \tag{6}$$

$$\bar{\phi}\gamma^\mu\partial_\mu\phi = \bar{\phi}_a(\gamma^\mu)^a{}_b(\partial_\mu\phi)^b = -(\partial_\mu\phi)^b(\gamma^\mu)^a{}_b\bar{\phi}_a = -(\partial_\mu\phi)^T(\gamma^\mu)^T\bar{\phi}^T \tag{7}$$

$$\bar{\phi}\phi = \bar{\phi}_a\phi^a = -\phi^a\bar{\phi}_a = -\phi^T\bar{\phi}^T \tag{8}$$

$$\int d^4x\bar{\phi}(i\gamma^\mu\partial_\mu - m)\phi = i\int d^4x\bar{\phi}\gamma^\mu\partial_\mu\phi - m\int d^4x\bar{\phi}\phi \tag{9}$$

$$= -i\int d^4x(\partial_\mu\phi)^T(\gamma^\mu)^T\bar{\phi}^T + m\int d^4x\phi^T\bar{\phi}^T \tag{10}$$

$$= i\int d^4x\phi^T(\gamma^\mu)^T\partial_\mu\bar{\phi}^T + m\int d^4x\phi^T\bar{\phi}^T \tag{11}$$

$$= -i\int d^4x\bar{\phi^C}C(\gamma^\mu)^T\partial_\mu C^{-1}\phi^C - m\int d^4x\bar{\phi^C}CC^{-1}\phi^C \tag{12}$$

$$= i\int d^4x\bar{\phi^C}\gamma^\mu\partial_\mu\phi^C - m\int d^4x\bar{\phi^C}\phi^C \tag{13}$$

$$= \int d^4x\bar{\phi^C}(i\gamma^\mu\partial_\mu - m)\phi^C \tag{14}$$

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)