場の量子論 p359check12.3

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2025 年 2 月 13 日

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$$
\begin{aligned}
&\mathscr{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi && (1)\\
&\psi(x) \to \psi'(x) \equiv \psi(x+\epsilon) = \psi(x) + \epsilon_\nu\partial^\nu\psi(x) \equiv \psi(x) + \delta_P\psi(x) && (2)\\
&\delta_P\mathscr{L} \equiv \mathscr{L}(\psi(x)+\delta_P\psi(x), \partial_\mu\{\psi(x)+\delta_P\psi(x)\}) - \mathscr{L}(\psi(x), \partial_\mu\psi(x)) && (3)\\
&\quad = \mathscr{L}(\psi(x+\epsilon), \partial_\mu\psi(x+\epsilon)) - \mathscr{L}(\psi(x), \partial_\mu\psi(x)) && (4)\\
&\quad = \mathscr{L}(x+\epsilon) - \mathscr{L}(x) = \epsilon_\nu\partial^\nu\mathscr{L}(x) = \partial_\mu(\epsilon_\nu\eta^{\mu\nu}\mathscr{L}) && (5)\\
&N^\mu \equiv \frac{\partial\mathscr{L}}{\partial(\partial_\mu\psi)}\delta_P\psi - (\epsilon_\nu\eta^{\mu\nu}\mathscr{L}) = \bar{\psi}i\gamma^\mu(\epsilon_\nu\partial^\nu\psi) - (\epsilon_\nu\eta^{\mu\nu}\mathscr{L}) && (6)\\
&N^0 = \epsilon_\nu(\psi^\dagger\gamma^0 i\gamma^0\partial^\nu\psi - \eta^{0\nu}\mathscr{L}) = \epsilon_\nu(i\psi^\dagger\partial^\nu\psi - \eta^{0\nu}\mathscr{L}) && (7)\\
&P^\nu = \int d^3\boldsymbol{x}(i\psi^\dagger\partial^\nu\psi - \eta^{0\nu}\mathscr{L}) && (8)
\end{aligned}
$$

$$\delta_J\psi(x) \equiv \psi'(x) - \psi(x) \tag{9}$$
$$= \psi'(x' - \Delta\omega \cdot x) - \psi(x) \tag{10}$$
$$= \psi'(x') - (\Delta\omega^{\mu}{}_{\nu}x^{\nu})\partial'_{\mu}\psi'(x') - \psi(x) \tag{11}$$
$$= (I_4 - \frac{i}{4}\Delta\omega_{\beta\nu}\sigma^{\beta\nu})\psi(x) - (\Delta\omega^{\mu}{}_{\nu}x^{\nu})\partial_{\mu}\psi(x) - \psi(x) \tag{12}$$
$$= -(\Delta\omega^{\mu}{}_{\nu}x^{\nu})\partial_{\mu}\psi(x) - \frac{i}{4}\Delta\omega_{\beta\nu}\sigma^{\beta\nu}\psi(x) \tag{13}$$
$$\delta_J\mathscr{L} \equiv \mathscr{L}(\psi(x) + \delta_J\psi(x)) - \mathscr{L}(\psi(x)) \tag{14}$$
$$= \mathscr{L}(x - \Delta\omega \cdot x) - \mathscr{L}(x) \tag{15}$$
$$= -(\Delta\omega^{\mu}{}_{\nu}x^{\nu})\partial_{\mu}\mathscr{L}(x) = \partial_{\mu}(-\Delta\omega^{\mu}{}_{\nu}x^{\nu}\mathscr{L}) \tag{16}$$
$$N^{\mu} \equiv \frac{\partial\mathscr{L}}{\partial(\partial_{\mu}\psi)}\delta_J\psi - (-\Delta\omega^{\mu}{}_{\nu}x^{\nu}\mathscr{L}) \tag{17}$$
$$= \bar{\psi}i\gamma^{\mu}[-(\Delta\omega^{\alpha}{}_{\nu}x^{\nu})\partial_{\alpha}\psi - \frac{i}{4}\Delta\omega_{\beta\nu}\sigma^{\beta\nu}\psi] - (-\Delta\omega^{\mu}{}_{\nu}x^{\nu}\mathscr{L}) \tag{18}$$
$$N^0 = -\psi^{\dagger}\gamma^0 i\gamma^0[(\Delta\omega^{\alpha}{}_{\nu}x^{\nu})\partial_{\alpha}\psi - \frac{i}{4}\Delta\omega_{\beta\nu}\sigma^{\beta\nu}\psi] + (\Delta\omega^{0}{}_{\nu}x^{\nu}\mathscr{L}) \tag{19}$$
$$= -i\psi^{\dagger}[(\Delta\omega^{\alpha}{}_{\nu}x^{\nu})\partial_{\alpha}\psi - \frac{i}{4}\Delta\omega_{\beta\nu}\sigma^{\beta\nu}\psi] + (\Delta\omega^{\alpha}{}_{\nu}\eta_{0\alpha}x^{\nu}\mathscr{L}) \tag{20}$$
$$= \Delta\omega^{\alpha}{}_{\nu}x^{\nu}[-i\psi^{\dagger}\partial_{\alpha}\psi + \eta_{0\alpha}\mathscr{L}] - \frac{1}{4}\psi^{\dagger}\Delta\omega_{\beta\nu}\sigma^{\beta\nu}\psi \tag{21}$$
$$= \Delta\omega_{\beta\nu}\eta^{\beta\alpha}x^{\nu}[-i\psi^{\dagger}\partial_{\alpha}\psi + \eta_{0\alpha}\mathscr{L}] - \frac{1}{4}\psi^{\dagger}\Delta\omega_{\beta\nu}\sigma^{\beta\nu}\psi \tag{22}$$
$$= \Delta\omega_{\beta\nu}x^{\nu}[-i\psi^{\dagger}\partial^{\beta}\psi + \delta^{0\beta}\mathscr{L}] - \frac{1}{4}\psi^{\dagger}\Delta\omega_{\beta\nu}\sigma^{\beta\nu}\psi \tag{23}$$
$$= \frac{1}{2}\Delta\omega_{\beta\nu}[-i\psi^{\dagger}x^{\nu}\partial^{\beta}\psi + \delta^{0\beta}x^{\nu}\mathscr{L} + i\psi^{\dagger}x^{\beta}\partial^{\nu}\psi - \delta^{0\nu}x^{\beta}\mathscr{L} - \frac{1}{2}\psi^{\dagger}\sigma^{\beta\nu}\psi] \tag{24}$$
$$= \frac{i}{2}\Delta\omega_{\beta\nu}[\psi^{\dagger}(x^{\beta}\partial^{\nu} - x^{\nu}\partial^{\beta} + \frac{i}{2}\sigma^{\beta\nu})\psi - i\delta^{0\beta}x^{\nu}\mathscr{L} + i\delta^{0\nu}x^{\beta}\mathscr{L}] \tag{25}$$
$$J^{ij} = \int d^3\boldsymbol{x}\psi^{\dagger}[i(x^i\partial^j - x^j\partial^i) + \frac{1}{2}\sigma^{ij}]\psi \tag{26}$$

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)