場の量子論 p361check12.4

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$$
\begin{aligned}
&[P^\mu, \psi^a(x)] && (1)\\
&= [\int d^3\boldsymbol{y} \left(i\psi_b^\dagger(t,\boldsymbol{y})\partial^\mu\psi^b(t,\boldsymbol{y}) - \eta^{0\mu}\mathscr{L}(t,\boldsymbol{y})\right), \psi^a(t,\boldsymbol{x})] && (2)\\
&= [\int d^3\boldsymbol{y} \left(i\psi_b^\dagger(t,\boldsymbol{y})\partial^\mu\psi^b(t,\boldsymbol{y}) - \eta^{0\mu}\psi_b^\dagger(t,\boldsymbol{y})[\gamma^0(i\gamma^\mu\partial_\mu - m)]^b{}_c\psi^c(t,\boldsymbol{y})\right), \psi^a(t,\boldsymbol{x})] && (3)\\
&= -i\int d^3\boldsymbol{y}\{\psi_b^\dagger(t,\boldsymbol{y}),\psi^a(t,\boldsymbol{x})\}\partial^\mu\psi^b(t,\boldsymbol{y}) + i\int d^3\boldsymbol{y}\psi_b^\dagger(t,\boldsymbol{y})\partial^\mu\{\psi^b(t,\boldsymbol{y}),\psi^a(t,\boldsymbol{x})\}\\
&\quad + \eta^{0\mu}\int d^3\boldsymbol{y}\{\psi_b^\dagger(t,\boldsymbol{y}),\psi^a(t,\boldsymbol{x})\}[\gamma^0(i\gamma^\mu\partial_\mu - m)]^b{}_c\psi^c(t,\boldsymbol{y})\\
&\quad - \eta^{0\mu}\int d^3\boldsymbol{y}\psi_b^\dagger(t,\boldsymbol{y})[\gamma^0(i\gamma^\mu\partial_\mu - m)]^b{}_c\{\psi^c(t,\boldsymbol{y}),\psi^a(t,\boldsymbol{x})\} && (4)\\
&= -i\int d^3\boldsymbol{y}\delta_b{}^a\delta(\boldsymbol{y}-\boldsymbol{x})\partial^\mu\psi^b(t,\boldsymbol{y}) + \eta^{0\mu}\int d^3\boldsymbol{y}\delta_b{}^a\delta(\boldsymbol{y}-\boldsymbol{x})[\gamma^0(i\gamma^\mu\partial_\mu - m)]^b{}_c\psi^c(t,\boldsymbol{y}) && (5)\\
&= -i\partial^\mu\psi^a(t,\boldsymbol{x}) + \eta^{0\mu}[\gamma^0(i\gamma^\mu\partial_\mu - m)\psi(t,\boldsymbol{x})]^a && (6)\\
&= -i\partial^\mu\psi^a(x) && (7)
\end{aligned}
$$

$$
\begin{aligned}
&[J^{ij}, \psi^a(x)] && (8)\\
&= [\int d^3\boldsymbol{y}\psi_b^\dagger(t,\boldsymbol{y})i(y^i\partial_y^j - y^j\partial_y^i)\psi^b(t,\boldsymbol{y}), \psi^a(t,\boldsymbol{x})] + [\int d^3\boldsymbol{y}\psi_b^\dagger(t,\boldsymbol{y})(\frac{1}{2}\sigma^{ij})^b{}_c\psi^c(t,\boldsymbol{y}), \psi^a(t,\boldsymbol{x})] && (9)\\
&= -\int d^3\boldsymbol{y}\{\psi_b^\dagger(t,\boldsymbol{y}),\psi^a(t,\boldsymbol{x})\}i(y^i\partial_y^j - y^j\partial_y^i)\psi^b(t,\boldsymbol{y}) + \int d^3\boldsymbol{y}\psi_b^\dagger(t,\boldsymbol{y})i(y^i\partial_y^j - y^j\partial_y^i)\{\psi^b(t,\boldsymbol{y}),\psi^a(t,\boldsymbol{x})\}\\
&\quad - \int d^3\boldsymbol{y}\{\psi_b^\dagger(t,\boldsymbol{y}),\psi^a(t,\boldsymbol{x})\}(\frac{1}{2}\sigma^{ij})^b{}_c\psi^c(t,\boldsymbol{y}) + \int d^3\boldsymbol{y}\psi_b^\dagger(t,\boldsymbol{y})(\frac{1}{2}\sigma^{ij})^b{}_c\{\psi^c(t,\boldsymbol{y}),\psi^a(t,\boldsymbol{x})\} && (10)\\
&= -\int d^3\boldsymbol{y}\delta_b{}^a\delta(\boldsymbol{y}-\boldsymbol{x})i(y^i\partial_y^j - y^j\partial_y^i)\psi^b(t,\boldsymbol{y}) - \int d^3\boldsymbol{y}\delta_b{}^a\delta(\boldsymbol{y}-\boldsymbol{x})(\frac{1}{2}\sigma^{ij})^b{}_c\psi^c(t,\boldsymbol{y}) && (11)\\
&= -\left[\left(i(y^i\partial^j - y^j\partial^i) + (\frac{1}{2}\sigma^{ij})\right)\psi(x)\right]^a && (12)
\end{aligned}
$$

$$[Q, \psi^a(x)] \tag{13}$$
$$= [q \int d^3\boldsymbol{y} \psi_b^\dagger(t,\boldsymbol{y}) \psi^b(t,\boldsymbol{y}), \psi^a(t,\boldsymbol{x})] \tag{14}$$
$$= -q \int d^3\boldsymbol{y} \{\psi_b^\dagger(t,\boldsymbol{y}), \psi^a(t,\boldsymbol{x})\} \psi^b(t,\boldsymbol{y}) + q \int d^3\boldsymbol{y} \psi_b^\dagger(t,\boldsymbol{y}) \{\psi^b(t,\boldsymbol{y}), \psi^a(t,\boldsymbol{x})\} \tag{15}$$
$$= -q \int d^3\boldsymbol{y} \delta_b{}^a \delta(\boldsymbol{y}-\boldsymbol{x}) \psi^b(t,\boldsymbol{y}) \tag{16}$$
$$= -q\psi^a(x) \tag{17}$$

$$[Q, \psi_a^\dagger(x)] \tag{18}$$
$$= [q \int d^3\boldsymbol{y} \psi_b^\dagger(t,\boldsymbol{y}) \psi^b(t,\boldsymbol{y}), \psi_a^\dagger(t,\boldsymbol{x})] \tag{19}$$
$$= -q \int d^3\boldsymbol{y} \{\psi_b^\dagger(t,\boldsymbol{y}), \psi_a^\dagger(t,\boldsymbol{x})\} \psi^b(t,\boldsymbol{y}) + q \int d^3\boldsymbol{y} \psi_b^\dagger(t,\boldsymbol{y}) \{\psi^b(t,\boldsymbol{y}), \psi_a^\dagger(t,\boldsymbol{x})\} \tag{20}$$
$$= +q \int d^3\boldsymbol{y} \psi_b^\dagger(t,\boldsymbol{y}) \delta^b{}_a \delta(\boldsymbol{y}-\boldsymbol{x}) \tag{21}$$
$$= +q\psi_a^\dagger(x) \tag{22}$$

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)