場の量子論 p366check12.5

谷脇 貞善

2025 年 3 月 9 日

1　本文

$$\sigma^1\sigma^2 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} = \begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix} = i\sigma^3 \tag{1}$$

$$\gamma^0 = \gamma^0_D = \begin{pmatrix}I_2 & \mathbf{0}\\ \mathbf{0} & -I_2\end{pmatrix}, \quad \gamma^j = \gamma^j_D = \begin{pmatrix}\mathbf{0} & \sigma^j\\ -\sigma^j & \mathbf{0}\end{pmatrix} \tag{2}$$

$$C = i\gamma^2\gamma^0 = i\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & i & 0\\ 0 & i & 0 & 0\\ -i & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix} \tag{3}$$

$$= \begin{pmatrix}0 & 0 & 0 & 1\\ 0 & 0 & -1 & 0\\ 0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix} = \begin{pmatrix}0 & 0 & 0 & -1\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\end{pmatrix} \tag{4}$$

$$\tilde{\not{k}} = \tilde{k}_\mu\gamma^\mu = m\gamma^0 = \begin{pmatrix}m & 0 & 0 & 0\\ 0 & m & 0 & 0\\ 0 & 0 & -m & 0\\ 0 & 0 & 0 & -m\end{pmatrix} \tag{5}$$

$$\tilde{\not{k}} - m = \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -2m & 0\\ 0 & 0 & 0 & -2m\end{pmatrix}, \quad \tilde{\not{k}} + m = \begin{pmatrix}2m & 0 & 0 & 0\\ 0 & 2m & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} \tag{6}$$

$$\frac{\tilde{\not{k}} - m}{2m} = \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}, \quad \frac{\tilde{\not{k}} + m}{2m} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} \tag{7}$$

$$(\tilde{\not{k}} - m)\tilde{u}(\tilde{\boldsymbol{k}} = 0, s) = \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -2m & 0\\ 0 & 0 & 0 & -2m\end{pmatrix}\begin{pmatrix}\tilde{u}_1\\ \tilde{u}_2\\ \tilde{u}_3\\ \tilde{u}_4\end{pmatrix} = 0 \tag{8}$$

$$\tilde{u}_3 = \tilde{u}_4 = 0 \tag{9}$$

$$\tilde{u}(\tilde{\boldsymbol{k}}=0,+)=\begin{pmatrix}\sqrt{2m}\\0\\0\\0\end{pmatrix}$$

$$\tilde{u}(\tilde{\boldsymbol{k}}=0,-)=\begin{pmatrix}0\\\sqrt{2m}\\0\\0\end{pmatrix} \tag{10}$$

$$\bar{\tilde{u}}(\tilde{\boldsymbol{k}}=0,+)=\tilde{u}^\dagger(\tilde{\boldsymbol{k}}=0,+)\gamma^0=\begin{pmatrix}\sqrt{2m}&0&0&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}=\begin{pmatrix}\sqrt{2m}&0&0&0\end{pmatrix}$$

$$\bar{\tilde{u}}(\tilde{\boldsymbol{k}}=0,-)=\tilde{u}^\dagger(\tilde{\boldsymbol{k}}=0,-)\gamma^0=\begin{pmatrix}0&\sqrt{2m}&0&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}=\begin{pmatrix}0&\sqrt{2m}&0&0\end{pmatrix} \tag{11}$$

$$\bar{\tilde{u}}(\tilde{\boldsymbol{k}}=0,s)\tilde{u}(\tilde{\boldsymbol{k}}=0,s')=2m\delta_{ss'} \tag{12}$$

$$\tilde{v}(\tilde{\boldsymbol{k}}=0,+)=C\bar{\tilde{u}}^T(\tilde{\boldsymbol{k}}=0,+)=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix}\begin{pmatrix}\sqrt{2m}\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\0\\\sqrt{2m}\end{pmatrix}$$

$$\tilde{v}(\tilde{\boldsymbol{k}}=0,-)=C\bar{\tilde{u}}^T(\tilde{\boldsymbol{k}}=0,-)=\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix}\begin{pmatrix}0\\\sqrt{2m}\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\-\sqrt{2m}\\0\end{pmatrix} \tag{13}$$

$$\bar{\tilde{v}}(\tilde{\boldsymbol{k}}=0,+)=\tilde{v}^\dagger(\tilde{\boldsymbol{k}}=0,+)\gamma^0=\begin{pmatrix}0&0&0&\sqrt{2m}\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}=\begin{pmatrix}0&0&0&-\sqrt{2m}\end{pmatrix}$$

$$\bar{\tilde{v}}(\tilde{\boldsymbol{k}}=0,-)=\tilde{v}^\dagger(\tilde{\boldsymbol{k}}=0,-)\gamma^0=\begin{pmatrix}0&0&-\sqrt{2m}&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}=\begin{pmatrix}0&0&\sqrt{2m}&0\end{pmatrix} \tag{14}$$

$$\bar{\tilde{v}}(\tilde{\boldsymbol{k}}=0,s)\tilde{v}(\tilde{\boldsymbol{k}}=0,s')=-2m\delta_{ss'} \tag{15}$$

$$\bar{\tilde{u}}(\tilde{\boldsymbol{k}}=0,s)\tilde{v}(\tilde{\boldsymbol{k}}=0,s')=\bar{\tilde{v}}(\tilde{\boldsymbol{k}}=0,s)\tilde{u}(\tilde{\boldsymbol{k}}=0,s')=0 \tag{16}$$

$$\tilde{u}^\dagger(\tilde{\boldsymbol{k}}=0,s)\tilde{u}(\tilde{\boldsymbol{k}}=0,s')=\tilde{v}^\dagger(\tilde{\boldsymbol{k}}=0,s)\tilde{v}(\tilde{\boldsymbol{k}}=0,s')=2m\delta_{ss'}=2E_{\tilde{k}=0}\delta_{ss'} \tag{17}$$

$$\tilde{u}^\dagger(\tilde{\boldsymbol{k}}=0,s)\tilde{v}(-\tilde{\boldsymbol{k}}=0,s')=\tilde{v}^\dagger(\tilde{\boldsymbol{k}}=0,s)\tilde{u}(-\tilde{\boldsymbol{k}}=0,s')=0 \tag{18}$$

$$
\begin{aligned}
\bar{\tilde{u}}(\tilde{\boldsymbol{k}}=0,+)\gamma^1\tilde{u}(\tilde{\boldsymbol{k}}=0,+) &= \begin{pmatrix}\sqrt{2m} & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ -1 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\sqrt{2m}\\ 0\\ 0\\ 0\end{pmatrix}\\
&= \begin{pmatrix}\sqrt{2m} & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}0\\ 0\\ 0\\ -\sqrt{2m}\end{pmatrix} = 0
\end{aligned} \tag{19}
$$

$$
\bar{\tilde{u}}(\tilde{\boldsymbol{k}}=0,s)\gamma^\mu\tilde{u}(\tilde{\boldsymbol{k}}=0,s') = \bar{\tilde{v}}(\tilde{\boldsymbol{k}}=0,s)\gamma^\mu\tilde{v}(\tilde{\boldsymbol{k}}=0,s') = \tilde{k}^\mu\delta_{ss'} \tag{20}
$$

$$
\begin{aligned}
\frac{1}{2m}\sum_{s=\pm}\tilde{u}^a(\tilde{\boldsymbol{k}}=0,s)\bar{\tilde{u}}_b(\tilde{\boldsymbol{k}}=0,s) &= \frac{1}{2m}\left[\begin{pmatrix}\sqrt{2m}\\ 0\\ 0\\ 0\end{pmatrix}\begin{pmatrix}\sqrt{2m} & 0 & 0 & 0\end{pmatrix} + \begin{pmatrix}0\\ \sqrt{2m}\\ 0\\ 0\end{pmatrix}\begin{pmatrix}0 & \sqrt{2m} & 0 & 0\end{pmatrix}\right]\\
&= \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} = \left(\frac{\tilde{\not{k}}+m}{2m}\right)^a{}_b
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\frac{1}{2m}\sum_{s=\pm}\tilde{v}^a(\tilde{\boldsymbol{k}}=0,s)\bar{\tilde{v}}_b(\tilde{\boldsymbol{k}}=0,s) &= \frac{1}{2m}\left[\begin{pmatrix}0\\ 0\\ 0\\ \sqrt{2m}\end{pmatrix}\begin{pmatrix}0 & 0 & 0 & -\sqrt{2m}\end{pmatrix} + \begin{pmatrix}0\\ 0\\ -\sqrt{2m}\\ 0\end{pmatrix}\begin{pmatrix}0 & 0 & \sqrt{2m} & 0\end{pmatrix}\right]\\
&= \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix} = \left(\frac{\tilde{\not{k}}-m}{2m}\right)^a{}_b
\end{aligned} \tag{22}
$$

$$
\frac{1}{2m}\sum_{s=\pm}\tilde{u}^a(\tilde{\boldsymbol{k}}=0,s)\bar{\tilde{u}}_b(\tilde{\boldsymbol{k}}=0,s) - \frac{1}{2m}\sum_{s=\pm}\tilde{v}^a(\tilde{\boldsymbol{k}}=0,s)\bar{\tilde{v}}_b(\tilde{\boldsymbol{k}}=0,s) = \delta^a{}_b \tag{23}
$$

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)