場の量子論 p368check12.6

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1　本文

$$\sigma^1\sigma^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = i\sigma^3, \quad \tilde{\chi}^{(s)} = -i\sigma^2(\chi^{(s)})^* \tag{1}$$

$$u(\boldsymbol{k}, s) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}} + m}\chi^{(s)} \\ \sqrt{E_{\boldsymbol{k}} - m}\frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{|\boldsymbol{k}|}\chi^{(s)} \end{pmatrix}, \quad v(\boldsymbol{k}, s) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}} - m}\frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{|\boldsymbol{k}|}\tilde{\chi}^{(s)} \\ \sqrt{E_{\boldsymbol{k}} + m}\tilde{\chi}^{(s)} \end{pmatrix} \tag{2}$$

$$\chi^{(s=+\frac{1}{2})} = \begin{pmatrix} e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}, \quad \chi^{(s=-\frac{1}{2})} = \begin{pmatrix} -e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \tag{3}$$

$$\boldsymbol{k} = (k_x, k_y, k_z) = (|\boldsymbol{k}|\cos\varphi\sin\theta, |\boldsymbol{k}|\sin\varphi\sin\theta, |\boldsymbol{k}|\cos\theta) \tag{4}$$

$$\frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{|\boldsymbol{k}|} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\cos\varphi\sin\theta + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\sin\varphi\sin\theta + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cos\theta = \begin{pmatrix} \cos\theta & e^{-i\varphi}\sin\theta \\ e^{+i\varphi}\sin\theta & -\cos\theta \end{pmatrix} \tag{5}$$

$$\gamma^0 = \gamma^0_D = \begin{pmatrix} I_2 & \mathbf{0} \\ \mathbf{0} & -I_2 \end{pmatrix}, \quad \gamma^j = \gamma^j_D = \begin{pmatrix} \mathbf{0} & \sigma^j \\ -\sigma^j & \mathbf{0} \end{pmatrix} \tag{6}$$

$$\begin{aligned} \not{k} = k_\mu\gamma^\mu = E_{\boldsymbol{k}} &\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} - |\boldsymbol{k}|\cos\varphi\sin\theta\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \\ &- |\boldsymbol{k}|\sin\varphi\sin\theta\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} - |\boldsymbol{k}|\cos\theta\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{aligned} \tag{7}$$

$$= \begin{pmatrix} E_{\boldsymbol{k}} & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ 0 & E_{\boldsymbol{k}} & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -E_{\boldsymbol{k}} & 0 \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -E_{\boldsymbol{k}} \end{pmatrix} \tag{8}$$

$$\not{k} + m = \begin{pmatrix} E_{\boldsymbol{k}} + m & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ 0 & E_{\boldsymbol{k}} + m & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -E_{\boldsymbol{k}} + m & 0 \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -E_{\boldsymbol{k}} + m \end{pmatrix} \tag{9}$$

$$\not{k} - m = \begin{pmatrix} E_{\boldsymbol{k}} - m & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ 0 & E_{\boldsymbol{k}} - m & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -E_{\boldsymbol{k}} - m & 0 \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -E_{\boldsymbol{k}} - m \end{pmatrix} \tag{10}$$

$$|\boldsymbol{k}| = \sqrt{E_{\boldsymbol{k}}^2 - m^2} = \sqrt{E_{\boldsymbol{k}} + m}\sqrt{E_{\boldsymbol{k}} - m} \tag{11}$$

$$u(\boldsymbol{k}, +\frac{1}{2}) \\ = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}} + m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} + m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} - m}[\cos\theta e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} + e^{-i\varphi}\sin\theta e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}] \\ \sqrt{E_{\boldsymbol{k}} - m}[e^{+i\varphi}\sin\theta e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} - \cos\theta e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}] \end{pmatrix} = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}} + m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} + m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} - m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} - m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \tag{12}$$

$$(\not{k} - m)u(\boldsymbol{k}, +\frac{1}{2}) \\ = \begin{pmatrix} E_{\boldsymbol{k}} - m & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ 0 & E_{\boldsymbol{k}} - m & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -E_{\boldsymbol{k}} - m & 0 \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -E_{\boldsymbol{k}} - m \end{pmatrix} \begin{pmatrix} \sqrt{E_{\boldsymbol{k}} + m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} + m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} - m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} - m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \tag{13}$$

$$= \begin{pmatrix} (E_{\boldsymbol{k}} - m)\sqrt{E_{\boldsymbol{k}} + m}e^{-\frac{i\varphi}{2}}[\cos\frac{\theta}{2} - \cos\theta\cos\frac{\theta}{2} - \sin\theta\sin\frac{\theta}{2}] \\ (E_{\boldsymbol{k}} - m)\sqrt{E_{\boldsymbol{k}} + m}e^{+\frac{i\varphi}{2}}[\sin\frac{\theta}{2} - \sin\theta\cos\frac{\theta}{2} + \cos\theta\sin\frac{\theta}{2}] \\ (E_{\boldsymbol{k}} + m)\sqrt{E_{\boldsymbol{k}} - m}e^{-\frac{i\varphi}{2}}[\cos\theta\cos\frac{\theta}{2} + \sin\theta\sin\frac{\theta}{2} - \cos\frac{\theta}{2}] \\ (E_{\boldsymbol{k}} + m)\sqrt{E_{\boldsymbol{k}} - m}e^{+\frac{i\varphi}{2}}[\sin\theta\cos\frac{\theta}{2} - \cos\theta\sin\frac{\theta}{2} - \sin\frac{\theta}{2}] \end{pmatrix} = 0 \tag{14}$$

$$u(\boldsymbol{k}, -\frac{1}{2}) \\ = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}} + m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} + m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} - m}[-\cos\theta e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} + e^{-i\varphi}\sin\theta e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}] \\ \sqrt{E_{\boldsymbol{k}} - m}[-e^{+i\varphi}\sin\theta e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} - \cos\theta e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}] \end{pmatrix} = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}} + m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} + m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} - m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}} - m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \tag{15}$$

$$(\not{k} - m)u(\boldsymbol{k}, -\frac{1}{2}) \\ = \begin{pmatrix} E_{\boldsymbol{k}} - m & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ 0 & E_{\boldsymbol{k}} - m & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -E_{\boldsymbol{k}} - m & 0 \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -E_{\boldsymbol{k}} - m \end{pmatrix} \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}} + m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} + m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}} - m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}} - m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \tag{16}$$

$$= \begin{pmatrix} (E_{\boldsymbol{k}} - m)\sqrt{E_{\boldsymbol{k}} + m}e^{-\frac{i\varphi}{2}}[-\sin\frac{\theta}{2} - \cos\theta\sin\frac{\theta}{2} + \sin\theta\cos\frac{\theta}{2}] \\ (E_{\boldsymbol{k}} - m)\sqrt{E_{\boldsymbol{k}} + m}e^{+\frac{i\varphi}{2}}[\cos\frac{\theta}{2} - \sin\theta\sin\frac{\theta}{2} - \cos\theta\cos\frac{\theta}{2}] \\ (E_{\boldsymbol{k}} + m)\sqrt{E_{\boldsymbol{k}} - m}e^{-\frac{i\varphi}{2}}[-\cos\theta\sin\frac{\theta}{2} + \sin\theta\cos\frac{\theta}{2} - \sin\frac{\theta}{2}] \\ (E_{\boldsymbol{k}} + m)\sqrt{E_{\boldsymbol{k}} - m}e^{+\frac{i\varphi}{2}}[-\sin\theta\sin\frac{\theta}{2} - \cos\theta\cos\frac{\theta}{2} + \cos\frac{\theta}{2}] \end{pmatrix} = 0 \tag{17}$$

$$v(\boldsymbol{k},s)=\begin{pmatrix}\sqrt{E_{\boldsymbol{k}}-m}\frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{|\boldsymbol{k}|}\tilde{\chi}^{(s)}\\ \sqrt{E_{\boldsymbol{k}}+m}\tilde{\chi}^{(s)}\end{pmatrix},\quad \frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{|\boldsymbol{k}|}=\begin{pmatrix}\cos\theta & e^{-i\varphi}\sin\theta\\ e^{+i\varphi}\sin\theta & -\cos\theta\end{pmatrix} \tag{18}$$

$$\tilde{\chi}^{(s=+\frac{1}{2})}=-i\sigma^2(\chi^{(s=+\frac{1}{2})})^*=-i\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}\\ e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2}\end{pmatrix}=\begin{pmatrix}-e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2}\\ e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}\end{pmatrix} \tag{19}$$

$$\begin{aligned}&v(\boldsymbol{k},s=+\frac{1}{2})\\ &=\begin{pmatrix}\sqrt{E_{\boldsymbol{k}}-m}[-\cos\theta e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2}+e^{-i\varphi}\sin\theta e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}]\\ \sqrt{E_{\boldsymbol{k}}-m}[-e^{+i\varphi}\sin\theta e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2}-\cos\theta e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}]\\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2}\\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}\end{pmatrix}=\begin{pmatrix}\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2}\\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}\end{pmatrix}\end{aligned} \tag{20}$$

$$\begin{aligned}&(\not{k}+m)v(\boldsymbol{k},s=+\frac{1}{2})\\ &=\begin{pmatrix}E_{\boldsymbol{k}}+m & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta\\ 0 & E_{\boldsymbol{k}}+m & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta\\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -E_{\boldsymbol{k}}+m & 0\\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -E_{\boldsymbol{k}}+m\end{pmatrix}\begin{pmatrix}\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2}\\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2}\end{pmatrix}\end{aligned} \tag{21}$$

$$=\begin{pmatrix}(E_{\boldsymbol{k}}+m)\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}[\sin\frac{\theta}{2}-\cos\theta\sin\frac{\theta}{2}-\sin\theta\cos\frac{\theta}{2}]\\ (E_{\boldsymbol{k}}+m)\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}[-\cos\frac{\theta}{2}+\sin\theta\sin\frac{\theta}{2}+\cos\theta\cos\frac{\theta}{2}]\\ (E_{\boldsymbol{k}}-m)\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}[\cos\theta\sin\frac{\theta}{2}+\sin\theta\cos\frac{\theta}{2}-\sin\frac{\theta}{2}]\\ (E_{\boldsymbol{k}}-m)\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}[\sin\theta\sin\frac{\theta}{2}+\cos\theta\cos\frac{\theta}{2}-\cos\frac{\theta}{2}]\end{pmatrix}=0 \tag{22}$$

$$\tilde{\chi}^{(s=-\frac{1}{2})}=-i\sigma^2(\chi^{(s=-\frac{1}{2})})^*=-i\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}-e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}\\ e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2}\end{pmatrix}=\begin{pmatrix}-e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2}\\ -e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}\end{pmatrix} \tag{23}$$

$$\begin{aligned}&v(\boldsymbol{k},s=-\frac{1}{2})\\ &=\begin{pmatrix}\sqrt{E_{\boldsymbol{k}}-m}[-\cos\theta e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2}-e^{-i\varphi}\sin\theta e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}]\\ \sqrt{E_{\boldsymbol{k}}-m}[-e^{+i\varphi}\sin\theta e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2}+\cos\theta e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}]\\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}\end{pmatrix}=\begin{pmatrix}-\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}\end{pmatrix}\end{aligned} \tag{24}$$

$$\begin{aligned}&(\not{k}+m)v(\boldsymbol{k},s=-\frac{1}{2})\\ &=\begin{pmatrix}E_{\boldsymbol{k}}+m & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta\\ 0 & E_{\boldsymbol{k}}+m & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta\\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -E_{\boldsymbol{k}}+m & 0\\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -E_{\boldsymbol{k}}+m\end{pmatrix}\begin{pmatrix}-\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2}\\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2}\end{pmatrix}\end{aligned} \tag{25}$$

$$=\begin{pmatrix}(E_{\boldsymbol{k}}+m)\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}[-\cos\frac{\theta}{2}+\cos\theta\cos\frac{\theta}{2}+\sin\theta\sin\frac{\theta}{2}]\\ (E_{\boldsymbol{k}}+m)\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}[-\sin\frac{\theta}{2}+\sin\theta\cos\frac{\theta}{2}-\cos\theta\sin\frac{\theta}{2}]\\ (E_{\boldsymbol{k}}-m)\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}[-\cos\theta\cos\frac{\theta}{2}-\sin\theta\sin\frac{\theta}{2}+\cos\frac{\theta}{2}]\\ (E_{\boldsymbol{k}}-m)\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}[-\sin\theta\cos\frac{\theta}{2}+\cos\theta\sin\frac{\theta}{2}+\sin\frac{\theta}{2}]\end{pmatrix}=0 \tag{26}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2}) = u^\dagger(\boldsymbol{k},+\frac{1}{2})\gamma^0 = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{27}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 2m \tag{28}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2}) = u^\dagger(\boldsymbol{k},-\frac{1}{2})\gamma^0 = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{29}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 2m \tag{30}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{31}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{32}$$

$$\bar{u}(\boldsymbol{k},s)u(\boldsymbol{k},s') = 2m\delta_{ss'} \tag{33}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2}) = v^\dagger(\boldsymbol{k},+\frac{1}{2})\gamma^0 = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{34}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = -2m \tag{35}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2}) = v^\dagger(\boldsymbol{k},-\frac{1}{2})\gamma^0 = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{36}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = -2m \tag{37}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{38}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{39}$$

$$\bar{v}(\boldsymbol{k},s)v(\boldsymbol{k},s') = -2m\delta_{ss'} \tag{40}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{41}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{42}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{43}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{44}$$

$$\bar{u}(\boldsymbol{k},s)v(\boldsymbol{k},s') = 0 \tag{45}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{46}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{47}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{48}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{49}$$

$$\bar{v}(\boldsymbol{k},s)u(\boldsymbol{k},s') = 0 \tag{50}$$

$$u^\dagger(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{51}$$

$$u^\dagger(\boldsymbol{k},+\frac{1}{2})u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 2E_{\boldsymbol{k}} \tag{52}$$

$$u^\dagger(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{53}$$

$$u^\dagger(\boldsymbol{k},-\frac{1}{2})u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 2E_{\boldsymbol{k}} \tag{54}$$

$$u^\dagger(\boldsymbol{k},+\frac{1}{2})u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{55}$$

$$u^\dagger(\boldsymbol{k},-\frac{1}{2})u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{56}$$

$$u^\dagger(\boldsymbol{k},s)u(\boldsymbol{k},s') = 2E_{\boldsymbol{k}}\delta_{ss'} \tag{57}$$

$$v^\dagger(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{58}$$

$$v^\dagger(\boldsymbol{k},+\frac{1}{2})v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = -2E_{\boldsymbol{k}} \tag{59}$$

$$v^\dagger(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{60}$$

$$v^\dagger(\boldsymbol{k},-\frac{1}{2})v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = -2E_{\boldsymbol{k}} \tag{61}$$

$$v^\dagger(\boldsymbol{k},+\frac{1}{2})v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{62}$$

$$v^\dagger(\boldsymbol{k},-\frac{1}{2})v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{63}$$

$$v^\dagger(\boldsymbol{k},s)v(\boldsymbol{k},s') = 2E_{\boldsymbol{k}}\delta_{ss'} \tag{64}$$

$$v(-\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta+\pi}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta+\pi}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta+\pi}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta+\pi}{2} \end{pmatrix} = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \tag{65}$$

$$v(-\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta+\pi}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta+\pi}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta+\pi}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta+\pi}{2} \end{pmatrix} = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \tag{66}$$

$$u^\dagger(\boldsymbol{k},+\frac{1}{2})v(-\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{67}$$

$$u^\dagger(\boldsymbol{k},+\frac{1}{2})v(-\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{68}$$

$$u^\dagger(\boldsymbol{k},-\frac{1}{2})v(-\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{69}$$

$$u^\dagger(\boldsymbol{k},-\frac{1}{2})v(-\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{70}$$

$$u^\dagger(\boldsymbol{k},s)v(-\boldsymbol{k},s') = 0 \tag{71}$$

$$v^\dagger(-\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{72}$$

$$v(-\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{73}$$

$$v^\dagger(-\boldsymbol{k},+\frac{1}{2})u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{74}$$

$$v^\dagger(-\boldsymbol{k},+\frac{1}{2})u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{75}$$

$$v^\dagger(-\boldsymbol{k},-\frac{1}{2})u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{76}$$

$$v^\dagger(-\boldsymbol{k},-\frac{1}{2})u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{77}$$

$$v^\dagger(-\boldsymbol{k},s)u(\boldsymbol{k},s') = 0 \tag{78}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})\gamma^1 = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{79}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})\gamma^1 u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \tag{80}$$

$$= |\boldsymbol{k}|(e^{+i\varphi}+e^{-i\varphi})2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2|\boldsymbol{k}|\cos\varphi\sin\theta \tag{81}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})\gamma^1 = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{82}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})\gamma^1 u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \tag{83}$$

$$= |\boldsymbol{k}|(e^{+i\varphi}+e^{-i\varphi})2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2|\boldsymbol{k}|\cos\varphi\sin\theta \tag{84}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})\gamma^1 u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{85}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})\gamma^1 u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{86}$$

$$\bar{u}(\boldsymbol{k},s)\gamma^1 u(\boldsymbol{k},s') = 2k_x\delta_{ss'} \tag{87}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})\gamma^1 = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{88}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})\gamma^1 v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \tag{89}$$

$$= |\boldsymbol{k}|(e^{+i\varphi}+e^{-i\varphi})2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2|\boldsymbol{k}|\cos\varphi\sin\theta \tag{90}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})\gamma^1 = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{91}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})\gamma^1 v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \tag{92}$$

$$= |\boldsymbol{k}|(e^{+i\varphi}+e^{-i\varphi})2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2|\boldsymbol{k}|\cos\varphi\sin\theta \tag{93}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})\gamma^1 v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{94}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})\gamma^1 v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{95}$$

$$\bar{v}(\boldsymbol{k},s)\gamma^1 v(\boldsymbol{k},s') = 2k_x\delta_{ss'} \tag{96}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})\gamma^2 = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{97}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})\gamma^2 u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \tag{98}$$

$$= |\boldsymbol{k}|[(-i)e^{+i\varphi} + ie^{-i\varphi}]2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2|\boldsymbol{k}|\sin\varphi\sin\theta \tag{99}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})\gamma^2 = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{100}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})\gamma^2 u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \tag{101}$$

$$= |\boldsymbol{k}|[(-i)e^{+i\varphi} + ie^{-i\varphi}]2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2|\boldsymbol{k}|\sin\varphi\sin\theta \tag{102}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})\gamma^2 u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{103}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})\gamma^2 u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{104}$$

$$\bar{u}(\boldsymbol{k},s)\gamma^2 u(\boldsymbol{k},s') = 2k_y\delta_{ss'} \tag{105}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})\gamma^2 = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{106}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})\gamma^2 v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \tag{107}$$

$$= |\boldsymbol{k}|[(-i)e^{+i\varphi} + ie^{-i\varphi}]2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2|\boldsymbol{k}|\sin\varphi\sin\theta \tag{108}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})\gamma^2 = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{109}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})\gamma^2 v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \tag{110}$$

$$= |\boldsymbol{k}|[(-i)e^{+i\varphi} + ie^{-i\varphi}]2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2|\boldsymbol{k}|\sin\varphi\sin\theta \tag{111}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})\gamma^2 v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{112}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})\gamma^2 v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -i\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -i\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ i\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{113}$$

$$\bar{v}(\boldsymbol{k},s)\gamma^2 v(\boldsymbol{k},s') = 2k_y\delta_{ss'} \tag{114}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})\gamma^3 = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{115}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})\gamma^3 u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \tag{116}$$

$$= |\boldsymbol{k}|2\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right) = 2|\boldsymbol{k}|\cos\theta \tag{117}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})\gamma^3 = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{118}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})\gamma^3 u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \tag{119}$$

$$= |\boldsymbol{k}|2\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right) = 2|\boldsymbol{k}|\cos\theta \tag{120}$$

$$\bar{u}(\boldsymbol{k},+\frac{1}{2})\gamma^3 u(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{121}$$

$$\bar{u}(\boldsymbol{k},-\frac{1}{2})\gamma^3 u(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{122}$$

$$\bar{u}(\boldsymbol{k},s)\gamma^3 u(\boldsymbol{k},s') = 2k_z\delta_{ss'} \tag{123}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})\gamma^3 = \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{124}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})\gamma^3 v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \tag{125}$$

$$= |\boldsymbol{k}|2\left(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right) = 2|\boldsymbol{k}|\cos\theta \tag{126}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})\gamma^3 = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \tag{127}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})\gamma^3 v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \tag{128}$$

$$= |\boldsymbol{k}|2\left(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right) = 2|\boldsymbol{k}|\cos\theta \tag{129}$$

$$\bar{v}(\boldsymbol{k},+\frac{1}{2})\gamma^3 v(\boldsymbol{k},-\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} = 0 \tag{130}$$

$$\bar{v}(\boldsymbol{k},-\frac{1}{2})\gamma^3 v(\boldsymbol{k},+\frac{1}{2}) = \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} = 0 \tag{131}$$

$$\bar{v}(\boldsymbol{k},s)\gamma^3 v(\boldsymbol{k},s') = 2k_z\delta_{ss'} \tag{132}$$

$$
\begin{aligned}
&\frac{1}{2m}u(\boldsymbol{k},+\frac{1}{2})\bar{u}(\boldsymbol{k},+\frac{1}{2}) \\
&= \frac{1}{2m}\begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T && (133) \\
&= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\cos^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ (E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\sin^2\frac{\theta}{2} \\ |\boldsymbol{k}|\cos^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\sin^2\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} \end{pmatrix} && (134)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1+(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2} \\
&= \frac{1}{2}\begin{pmatrix} 1+\boldsymbol{\sigma}\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|} & 0 \\ 0 & 1+\boldsymbol{\sigma}\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|} \end{pmatrix} && (135) \\
&= \frac{1}{2}\begin{pmatrix} 1+\cos\theta & e^{-i\varphi}\sin\theta & 0 & 0 \\ e^{+i\varphi}\sin\theta & 1-\cos\theta & 0 & 0 \\ 0 & 0 & 1+\cos\theta & e^{-i\varphi}\sin\theta \\ 0 & 0 & e^{+i\varphi}\sin\theta & 1-\cos\theta \end{pmatrix} && (136) \\
&= \begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & 0 & 0 \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} & 0 & 0 \\ 0 & 0 & \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ 0 & 0 & e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} && (137)
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\not{k}+m}{2m}\right)\left(\frac{1+(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) \\
&= \frac{1}{2m}\begin{pmatrix} E_{\boldsymbol{k}}+m & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ 0 & E_{\boldsymbol{k}}+m & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -E_{\boldsymbol{k}}+m & 0 \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -E_{\boldsymbol{k}}+m \end{pmatrix} \\
&\quad\times\begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & 0 & 0 \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} & 0 & 0 \\ 0 & 0 & \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ 0 & 0 & e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} && (138) \\
&= \begin{pmatrix} A & B \\ C & D \end{pmatrix} && (139)
\end{aligned}
$$

$$A = \frac{1}{2m}\begin{pmatrix} E_{\boldsymbol{k}}+m & 0 \\ 0 & E_{\boldsymbol{k}}+m \end{pmatrix}\begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} \tag{140}$$

$$= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ (E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} \end{pmatrix} \tag{141}$$

$$B = \frac{1}{2m}\begin{pmatrix} -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \end{pmatrix}\begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} \tag{142}$$

$$= \frac{1}{2m}\begin{pmatrix} -|\boldsymbol{k}|(\cos\theta\cos^2\frac{\theta}{2}+\sin\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}) & -|\boldsymbol{k}|e^{-i\varphi}(\cos\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}+\sin\theta\sin^2\frac{\theta}{2}) \\ |\boldsymbol{k}|e^{+i\varphi}(-\sin\theta\cos^2\frac{\theta}{2}+\cos\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}) & |\boldsymbol{k}|(-\sin\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}+\cos\theta\sin^2\frac{\theta}{2}) \end{pmatrix} \tag{143}$$

$$= \frac{1}{2m}\begin{pmatrix} -|\boldsymbol{k}|\cos^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\sin^2\frac{\theta}{2} \end{pmatrix} \tag{144}$$

$$C = \frac{1}{2m}\begin{pmatrix} |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta \end{pmatrix}\begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} \tag{145}$$

$$= \frac{1}{2m}\begin{pmatrix} |\boldsymbol{k}|\cos^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\sin^2\frac{\theta}{2} \end{pmatrix} \tag{146}$$

$$D = \frac{1}{2m}\begin{pmatrix} -E_{\boldsymbol{k}}+m & 0 \\ 0 & -E_{\boldsymbol{k}}+m \end{pmatrix}\begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} \tag{147}$$

$$= \frac{1}{2m}\begin{pmatrix} (-E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ (-E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} \end{pmatrix} \tag{148}$$

$$\left(\frac{\not{k}+m}{2m}\right)\left(\frac{1+(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\cos^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ (E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\sin^2\frac{\theta}{2} \\ |\boldsymbol{k}|\cos^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\sin^2\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} \end{pmatrix} \tag{149}$$

$$\frac{1}{2m}u(\boldsymbol{k},+\frac{1}{2})\bar{u}(\boldsymbol{k},+\frac{1}{2}) = \left(\frac{\not{k}+m}{2m}\right)\left(\frac{1+(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) \tag{150}$$

$$
\begin{aligned}
&\frac{1}{2m}u(\boldsymbol{k},-\frac{1}{2})\bar{u}(\boldsymbol{k},-\frac{1}{2}) \\
&= \frac{1}{2m}\begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \qquad (151) \\
&= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} & -(E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\sin^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -(E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\cos^2\frac{\theta}{2} \\ -|\boldsymbol{k}|\sin^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} & (E_{\boldsymbol{k}}-m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\cos^2\frac{\theta}{2} & (E_{\boldsymbol{k}}-m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} \end{pmatrix} \qquad (152)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1-(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2} \\
&= \frac{1}{2}\begin{pmatrix} 1-\boldsymbol{\sigma}\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|} & 0 \\ 0 & 1-\boldsymbol{\sigma}\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|} \end{pmatrix} \qquad (153) \\
&= \frac{1}{2}\begin{pmatrix} 1-\cos\theta & -e^{-i\varphi}\sin\theta & 0 & 0 \\ -e^{+i\varphi}\sin\theta & 1+\cos\theta & 0 & 0 \\ 0 & 0 & 1-\cos\theta & -e^{-i\varphi}\sin\theta \\ 0 & 0 & -e^{+i\varphi}\sin\theta & 1+\cos\theta \end{pmatrix} \qquad (154) \\
&= \begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & 0 & 0 \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} & 0 & 0 \\ 0 & 0 & \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ 0 & 0 & -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \qquad (155)
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\not{k}+m}{2m}\right)\left(\frac{1-(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) \\
&= \frac{1}{2m}\begin{pmatrix} E_{\boldsymbol{k}}+m & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ 0 & E_{\boldsymbol{k}}+m & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -E_{\boldsymbol{k}}+m & 0 \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -E_{\boldsymbol{k}}+m \end{pmatrix} \\
&\quad\times\begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & 0 & 0 \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} & 0 & 0 \\ 0 & 0 & \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ 0 & 0 & -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \qquad (156) \\
&= \begin{pmatrix} A & B \\ C & D \end{pmatrix} \qquad (157)
\end{aligned}
$$

$$A = \frac{1}{2m}\begin{pmatrix} E_{\boldsymbol{k}}+m & 0 \\ 0 & E_{\boldsymbol{k}}+m \end{pmatrix}\begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \tag{158}$$

$$= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} & -(E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -(E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} \end{pmatrix} \tag{159}$$

$$B = \frac{1}{2m}\begin{pmatrix} -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \end{pmatrix}\begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \tag{160}$$

$$= \frac{1}{2m}\begin{pmatrix} |\boldsymbol{k}|(-\cos\theta\sin^2\frac{\theta}{2}+\sin\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}) & |\boldsymbol{k}|e^{-i\varphi}(\cos\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}-\sin\theta\cos^2\frac{\theta}{2}) \\ -|\boldsymbol{k}|e^{+i\varphi}(\sin\theta\sin^2\frac{\theta}{2}+\cos\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}) & |\boldsymbol{k}|(\sin\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}+\cos\theta\cos^2\frac{\theta}{2}) \end{pmatrix} \tag{161}$$

$$= \frac{1}{2m}\begin{pmatrix} |\boldsymbol{k}|\sin^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\cos^2\frac{\theta}{2} \end{pmatrix} \tag{162}$$

$$C = \frac{1}{2m}\begin{pmatrix} |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta \end{pmatrix}\begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \tag{163}$$

$$= \frac{1}{2m}\begin{pmatrix} -|\boldsymbol{k}|\sin^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\cos^2\frac{\theta}{2} \end{pmatrix} \tag{164}$$

$$D = \frac{1}{2m}\begin{pmatrix} -E_{\boldsymbol{k}}+m & 0 \\ 0 & -E_{\boldsymbol{k}}+m \end{pmatrix}\begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \tag{165}$$

$$= \frac{1}{2m}\begin{pmatrix} (-E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} & (E_{\boldsymbol{k}}-m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ (E_{\boldsymbol{k}}-m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} \end{pmatrix} \tag{166}$$

$$\left(\frac{\not{k}+m}{2m}\right)\left(\frac{1-(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} & -(E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\sin^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -(E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\cos^2\frac{\theta}{2} \\ -|\boldsymbol{k}|\sin^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} & (E_{\boldsymbol{k}}-m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\cos^2\frac{\theta}{2} & (E_{\boldsymbol{k}}-m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (-E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} \end{pmatrix} \tag{167}$$

$$\frac{1}{2m}u(\boldsymbol{k},-\frac{1}{2})\bar{u}(\boldsymbol{k},-\frac{1}{2}) = \left(\frac{\not{k}+m}{2m}\right)\left(\frac{1-(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) \tag{168}$$

$$\frac{1}{2m}u(\boldsymbol{k},s)\bar{u}(\boldsymbol{k},s) = \left(\frac{\not{k}+m}{2m}\right)\left(\frac{1+2s(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) \tag{169}$$

$$\frac{1}{2m}\sum_{s=\pm\frac{1}{2}} u^a(\boldsymbol{k},s)\bar{u}_b(\boldsymbol{k},s) = \left(\frac{\not{k}+m}{2m}\right)^a_{\ b} \tag{170}$$

$$
\frac{1}{2m}v(\boldsymbol{k},+\frac{1}{2})\bar{v}(\boldsymbol{k},+\frac{1}{2})
$$

$$
= \frac{1}{2m}\begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \end{pmatrix}^T \tag{171}
$$

$$
= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}}-m)\sin^2\frac{\theta}{2} & -(E_{\boldsymbol{k}}-m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\sin^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -(E_{\boldsymbol{k}}-m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}}-m)\cos^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\cos^2\frac{\theta}{2} \\ -|\boldsymbol{k}|\sin^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -(E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\cos^2\frac{\theta}{2} & (E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -(E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} \end{pmatrix} \tag{172}
$$

$$
\left(\frac{\not{k}-m}{2m}\right)\left(\frac{1-(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right)
$$

$$
= \frac{1}{2m}\begin{pmatrix} E_{\boldsymbol{k}}-m & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ 0 & E_{\boldsymbol{k}}-m & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -(E_{\boldsymbol{k}}+m) & 0 \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -(E_{\boldsymbol{k}}+m) \end{pmatrix}
$$

$$
\times\begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & 0 & 0 \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} & 0 & 0 \\ 0 & 0 & \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ 0 & 0 & -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \tag{173}
$$

$$
= \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{174}
$$

$$A = \frac{1}{2m}\begin{pmatrix} E_{\boldsymbol{k}} - m & 0 \\ 0 & E_{\boldsymbol{k}} - m \end{pmatrix}\begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \tag{175}$$

$$= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}} - m)\sin^2\frac{\theta}{2} & -(E_{\boldsymbol{k}} - m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -(E_{\boldsymbol{k}} - m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}} - m)\cos^2\frac{\theta}{2} \end{pmatrix} \tag{176}$$

$$B = \frac{1}{2m}\begin{pmatrix} -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \end{pmatrix}\begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \tag{177}$$

$$= \frac{1}{2m}\begin{pmatrix} |\boldsymbol{k}|(-\cos\theta\sin^2\frac{\theta}{2} + \sin\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}) & |\boldsymbol{k}|e^{-i\varphi}(\cos\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \sin\theta\cos^2\frac{\theta}{2}) \\ -|\boldsymbol{k}|e^{+i\varphi}(\sin\theta\sin^2\frac{\theta}{2} + \cos\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}) & |\boldsymbol{k}|(\sin\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos\theta\cos^2\frac{\theta}{2}) \end{pmatrix} \tag{178}$$

$$= \frac{1}{2m}\begin{pmatrix} |\boldsymbol{k}|\sin^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\cos^2\frac{\theta}{2} \end{pmatrix} \tag{179}$$

$$C = \frac{1}{2m}\begin{pmatrix} |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta \end{pmatrix}\begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \tag{180}$$

$$= \frac{1}{2m}\begin{pmatrix} -|\boldsymbol{k}|\sin^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\cos^2\frac{\theta}{2} \end{pmatrix} \tag{181}$$

$$D = \frac{1}{2m}\begin{pmatrix} -(E_{\boldsymbol{k}} + m) & 0 \\ 0 & -(E_{\boldsymbol{k}} + m) \end{pmatrix}\begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix} \tag{182}$$

$$= \frac{1}{2m}\begin{pmatrix} -(E_{\boldsymbol{k}} + m)\sin^2\frac{\theta}{2} & (E_{\boldsymbol{k}} + m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ (E_{\boldsymbol{k}} + m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -(E_{\boldsymbol{k}} + m)\cos^2\frac{\theta}{2} \end{pmatrix} \tag{183}$$

$$\left(\frac{\not{k} + m}{2m}\right)\left(\frac{1 - (2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}} - m)\sin^2\frac{\theta}{2} & -(E_{\boldsymbol{k}} - m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\sin^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -(E_{\boldsymbol{k}} - m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}} - m)\cos^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\cos^2\frac{\theta}{2} \\ -|\boldsymbol{k}|\sin^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -(E_{\boldsymbol{k}} + m)\sin^2\frac{\theta}{2} & (E_{\boldsymbol{k}} + m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\cos^2\frac{\theta}{2} & (E_{\boldsymbol{k}} + m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -(E_{\boldsymbol{k}} + m)\cos^2\frac{\theta}{2} \end{pmatrix} \tag{184}$$

$$\frac{1}{2m}v(\boldsymbol{k}, +\frac{1}{2})\bar{v}(\boldsymbol{k}, +\frac{1}{2}) = \left(\frac{\not{k} - m}{2m}\right)\left(\frac{1 - (2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) \tag{185}$$

$$
\begin{aligned}
&\frac{1}{2m}v(\boldsymbol{k},-\frac{1}{2})\bar{v}(\boldsymbol{k},-\frac{1}{2}) \\
&= \frac{1}{2m}\begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} -\sqrt{E_{\boldsymbol{k}}-m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ -\sqrt{E_{\boldsymbol{k}}-m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{+\frac{i\varphi}{2}}\cos\frac{\theta}{2} \\ \sqrt{E_{\boldsymbol{k}}+m}e^{-\frac{i\varphi}{2}}\sin\frac{\theta}{2} \end{pmatrix}^T &(186)\\
&= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}}-m)\cos^2\frac{\theta}{2} & (E_{\boldsymbol{k}}-m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\cos^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ (E_{\boldsymbol{k}}-m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}}-m)\sin^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\sin^2\frac{\theta}{2} \\ |\boldsymbol{k}|\cos^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -(E_{\boldsymbol{k}}+m)\cos^2\frac{\theta}{2} & -(E_{\boldsymbol{k}}+m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\sin^2\frac{\theta}{2} & -(E_{\boldsymbol{k}}+m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -(E_{\boldsymbol{k}}+m)\sin^2\frac{\theta}{2} \end{pmatrix} &(187)
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\not{k}-m}{2m}\right)\left(\frac{1+(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) \\
&= \frac{1}{2m}\begin{pmatrix} E_{\boldsymbol{k}}-m & 0 & -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ 0 & E_{\boldsymbol{k}}-m & -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \\ |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta & -(E_{\boldsymbol{k}}+m) & 0 \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta & 0 & -(E_{\boldsymbol{k}}+m) \end{pmatrix} \\
&\qquad \times \begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & 0 & 0 \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} & 0 & 0 \\ 0 & 0 & \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ 0 & 0 & e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} &(188)\\
&= \begin{pmatrix} A & B \\ C & D \end{pmatrix} &(189)
\end{aligned}
$$

$$A = \frac{1}{2m}\begin{pmatrix} E_{\boldsymbol{k}} - m & 0 \\ 0 & E_{\boldsymbol{k}} - m \end{pmatrix}\begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} \tag{190}$$

$$= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}} - m)\cos^2\frac{\theta}{2} & (E_{\boldsymbol{k}} - m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ (E_{\boldsymbol{k}} - m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}} - m)\sin^2\frac{\theta}{2} \end{pmatrix} \tag{191}$$

$$B = \frac{1}{2m}\begin{pmatrix} -|\boldsymbol{k}|\cos\theta & -|\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ -|\boldsymbol{k}|e^{+i\varphi}\sin\theta & |\boldsymbol{k}|\cos\theta \end{pmatrix}\begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} \tag{192}$$

$$= \frac{1}{2m}\begin{pmatrix} -|\boldsymbol{k}|(\cos\theta\cos^2\frac{\theta}{2} + \sin\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}) & -|\boldsymbol{k}|e^{-i\varphi}(\cos\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \sin\theta\sin^2\frac{\theta}{2}) \\ |\boldsymbol{k}|e^{+i\varphi}(-\sin\theta\cos^2\frac{\theta}{2} + \cos\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2}) & |\boldsymbol{k}|(-\sin\theta\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos\theta\sin^2\frac{\theta}{2}) \end{pmatrix} \tag{193}$$

$$= \frac{1}{2m}\begin{pmatrix} -|\boldsymbol{k}|\cos^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\sin^2\frac{\theta}{2} \end{pmatrix} \tag{194}$$

$$C = \frac{1}{2m}\begin{pmatrix} |\boldsymbol{k}|\cos\theta & |\boldsymbol{k}|e^{-i\varphi}\sin\theta \\ |\boldsymbol{k}|e^{+i\varphi}\sin\theta & -|\boldsymbol{k}|\cos\theta \end{pmatrix}\begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} \tag{195}$$

$$= \frac{1}{2m}\begin{pmatrix} |\boldsymbol{k}|\cos^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\sin^2\frac{\theta}{2} \end{pmatrix} \tag{196}$$

$$D = \frac{1}{2m}\begin{pmatrix} -(E_{\boldsymbol{k}} + m) & 0 \\ 0 & -(E_{\boldsymbol{k}} + m) \end{pmatrix}\begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} \tag{197}$$

$$= \frac{1}{2m}\begin{pmatrix} -(E_{\boldsymbol{k}} + m)\cos^2\frac{\theta}{2} & -(E_{\boldsymbol{k}} + m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -(E_{\boldsymbol{k}} + m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -(E_{\boldsymbol{k}} + m)\sin^2\frac{\theta}{2} \end{pmatrix} \tag{198}$$

$$\begin{aligned} &\left(\frac{\not{k} - m}{2m}\right)\left(\frac{1 + (2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \\ &= \frac{1}{2m}\begin{pmatrix} (E_{\boldsymbol{k}} - m)\cos^2\frac{\theta}{2} & (E_{\boldsymbol{k}} - m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\cos^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ (E_{\boldsymbol{k}} - m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & (E_{\boldsymbol{k}} - m)\sin^2\frac{\theta}{2} & -|\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -|\boldsymbol{k}|\sin^2\frac{\theta}{2} \\ |\boldsymbol{k}|\cos^2\frac{\theta}{2} & |\boldsymbol{k}|e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -(E_{\boldsymbol{k}} + m)\cos^2\frac{\theta}{2} & -(E_{\boldsymbol{k}} + m)e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ |\boldsymbol{k}|e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & |\boldsymbol{k}|\sin^2\frac{\theta}{2} & -(E_{\boldsymbol{k}} + m)e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & -(E_{\boldsymbol{k}} + m)\sin^2\frac{\theta}{2} \end{pmatrix} \end{aligned} \tag{199}$$

$$\frac{1}{2m}v(\boldsymbol{k}, -\frac{1}{2})\bar{v}(\boldsymbol{k}, -\frac{1}{2}) = \left(\frac{\not{k} - m}{2m}\right)\left(\frac{1 + (2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) \tag{200}$$

$$\frac{1}{2m}v(\boldsymbol{k}, s)\bar{v}(\boldsymbol{k}, s) = \left(\frac{\not{k} - m}{2m}\right)\left(\frac{1 - 2s(2S\cdot\frac{\boldsymbol{k}}{|\boldsymbol{k}|})}{2}\right) \tag{201}$$

$$\frac{1}{2m}\sum_{s=\pm\frac{1}{2}} v^a(\boldsymbol{k}, s)\bar{v}_b(\boldsymbol{k}, s) = \left(\frac{\not{k} - m}{2m}\right)^a{}_b \tag{202}$$

$$\frac{1}{2m}\sum_{s=\pm\frac{1}{2}} \{u^a(\boldsymbol{k}, s)\bar{u}_b(\boldsymbol{k}, s) - v^a(\boldsymbol{k}, s)\bar{v}_b(\boldsymbol{k}, s)\} = \delta^a{}_b \tag{203}$$

参考文献

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