場の量子論 p369check12.7

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\begin{aligned}
(i\gamma^\mu\partial_\mu - m)\psi &= 0 && (1)\\
i\gamma^0\partial_0\psi &= -i\gamma^k\partial_k\psi + m\psi && (2)\\
\partial_0\psi &= -\gamma^0\gamma^k\partial_k\psi - im\gamma^0\psi && (3)\\
-i(\partial_\mu\bar{\psi}')\gamma^\mu - m\bar{\psi}' &= 0 && (4)\\
-i(\partial_0\bar{\psi}')\gamma^0 &= i(\partial_k\bar{\psi}')\gamma^k + m\bar{\psi}' && (5)\\
\partial_0\bar{\psi}' &= -(\partial_k\bar{\psi}')\gamma^k\gamma^0 + im\bar{\psi}'\gamma^0 && (6)\\
\partial_0(\psi'|\psi) &= \partial_0\int d^3\boldsymbol{x}\bar{\psi}'(x)\gamma^0\psi(x) && (7)\\
&= \int d^3\boldsymbol{x}\{[\partial_0\bar{\psi}'(x)]\gamma^0\psi(x) + \bar{\psi}'(x)\gamma^0[\partial_0\psi(x)]\} && (8)\\
&= \int d^3\boldsymbol{x}\{[-(\partial_k\bar{\psi}')\gamma^k\gamma^0 + im\bar{\psi}'\gamma^0]\gamma^0\psi + \bar{\psi}'\gamma^0[-\gamma^0\gamma^k\partial_k\psi - im\gamma^0\psi]\} && (9)\\
&= \int d^3\boldsymbol{x}\{-(\partial_k\bar{\psi}')\gamma^k\psi + im\bar{\psi}'\psi - \bar{\psi}'\gamma^k(\partial_k\psi) - im\bar{\psi}'\psi\} && (10)\\
&= -\int d^3\boldsymbol{x}\partial_k\{\bar{\psi}'\gamma^k\psi\} && (11)\\
&= 0 && (12)
\end{aligned}
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\begin{align}
U_{\boldsymbol{k},s}(x) &= \frac{u(\boldsymbol{k},s)e^{-ik\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \tag{13}\\
V_{\boldsymbol{k},s}(x) &= \frac{v(\boldsymbol{k},s)e^{+ik\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \tag{14}\\
(U_{\boldsymbol{k},s}|U_{\boldsymbol{k'},s'}) &= \int d^3\boldsymbol{x} U^\dagger_{\boldsymbol{k},s}(x) U_{\boldsymbol{k'},s'}(x) \tag{15}\\
&= \int d^3\boldsymbol{x} \frac{u^\dagger(\boldsymbol{k},s)e^{+ik\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \frac{u(\boldsymbol{k'},s')e^{-ik'\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} \tag{16}\\
&= \frac{u^\dagger(\boldsymbol{k},s)u(\boldsymbol{k'},s')}{(2\pi)^3 2\sqrt{E_{\boldsymbol{k}}E_{\boldsymbol{k'}}}} e^{+iE_{\boldsymbol{k}}t - iE_{\boldsymbol{k'}}t} \int d^3\boldsymbol{x} e^{-i\boldsymbol{k}\cdot\boldsymbol{x} + i\boldsymbol{k'}\cdot\boldsymbol{x}} \tag{17}\\
&= \frac{u^\dagger(\boldsymbol{k},s)u(\boldsymbol{k},s')}{(2\pi)^3 2E_{\boldsymbol{k}}} (2\pi)^3 \delta^3(\boldsymbol{k}-\boldsymbol{k'}) \tag{18}\\
&= \frac{2E_{\boldsymbol{k}}\delta_{ss'}}{2E_{\boldsymbol{k}}} \delta^3(\boldsymbol{k}-\boldsymbol{k'}) \tag{19}\\
&= \delta_{ss'}\delta^3(\boldsymbol{k}-\boldsymbol{k'}) \tag{20}\\
(V_{\boldsymbol{k},s}|V_{\boldsymbol{k'},s'}) &= \int d^3\boldsymbol{x} V^\dagger_{\boldsymbol{k},s}(x) V_{\boldsymbol{k'},s'}(x) \tag{21}\\
&= \int d^3\boldsymbol{x} \frac{v^\dagger(\boldsymbol{k},s)e^{-ik\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \frac{v(\boldsymbol{k'},s')e^{+ik'\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} \tag{22}\\
&= \frac{v^\dagger(\boldsymbol{k},s)v(\boldsymbol{k'},s')}{(2\pi)^3 2\sqrt{E_{\boldsymbol{k}}E_{\boldsymbol{k'}}}} e^{-iE_{\boldsymbol{k}}t + iE_{\boldsymbol{k'}}t} \int d^3\boldsymbol{x} e^{+i\boldsymbol{k}\cdot\boldsymbol{x} - i\boldsymbol{k'}\cdot\boldsymbol{x}} \tag{23}\\
&= \frac{v^\dagger(\boldsymbol{k},s)v(\boldsymbol{k},s')}{(2\pi)^3 2E_{\boldsymbol{k}}} (2\pi)^3 \delta^3(\boldsymbol{k}-\boldsymbol{k'}) \tag{24}\\
&= \frac{2E_{\boldsymbol{k}}\delta_{ss'}}{2E_{\boldsymbol{k}}} \delta^3(\boldsymbol{k}-\boldsymbol{k'}) \tag{25}\\
&= \delta_{ss'}\delta^3(\boldsymbol{k}-\boldsymbol{k'}) \tag{26}
\end{align}
$$

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\begin{aligned}
(U_{\boldsymbol{k},s}|V_{\boldsymbol{k'},s'}) &= \int d^3\boldsymbol{x} U^\dagger_{\boldsymbol{k},s}(x) V_{\boldsymbol{k'},s'}(x) && (27)\\
&= \int d^3\boldsymbol{x} \frac{u^\dagger(\boldsymbol{k},s)e^{+ik\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \frac{v(\boldsymbol{k'},s')e^{+ik'\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} && (28)\\
&= \frac{u^\dagger(\boldsymbol{k},s)v(\boldsymbol{k'},s')}{(2\pi)^3 2\sqrt{E_{\boldsymbol{k}}E_{\boldsymbol{k'}}}} e^{+iE_{\boldsymbol{k}}t+iE_{\boldsymbol{k'}}t} \int d^3\boldsymbol{x} e^{-i\boldsymbol{k}\cdot\boldsymbol{x}-i\boldsymbol{k'}\cdot\boldsymbol{x}} && (29)\\
&= \frac{u^\dagger(\boldsymbol{k},s)v(-\boldsymbol{k},s')}{(2\pi)^3 2E_{\boldsymbol{k}}} e^{+2iE_{\boldsymbol{k}}t}(2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k'}) && (30)\\
&= 0 && (31)\\
(V_{\boldsymbol{k},s}|U_{\boldsymbol{k'},s'}) &= \int d^3\boldsymbol{x} V^\dagger_{\boldsymbol{k},s}(x) U_{\boldsymbol{k'},s'}(x) && (32)\\
&= \int d^3\boldsymbol{x} \frac{v^\dagger(\boldsymbol{k},s)e^{-ik\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \frac{u(\boldsymbol{k'},s')e^{-ik'\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} && (33)\\
&= \frac{v^\dagger(\boldsymbol{k},s)u(\boldsymbol{k'},s')}{(2\pi)^3 2\sqrt{E_{\boldsymbol{k}}E_{\boldsymbol{k'}}}} e^{-iE_{\boldsymbol{k}}t-iE_{\boldsymbol{k'}}t} \int d^3\boldsymbol{x} e^{+i\boldsymbol{k}\cdot\boldsymbol{x}+i\boldsymbol{k'}\cdot\boldsymbol{x}} && (34)\\
&= \frac{v^\dagger(\boldsymbol{k},s)u(-\boldsymbol{k},s')}{(2\pi)^3 2E_{\boldsymbol{k}}} e^{-2iE_{\boldsymbol{k}}t}(2\pi)^3\delta^3(\boldsymbol{k}+\boldsymbol{k'}) && (35)\\
&= 0 && (36)
\end{aligned}
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参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)