場の量子論 p370check12.8

谷脇 貞善

2025 年 5 月 31 日

1 本文

$$U^a_{\boldsymbol{k},s}(x) = \frac{u^a(\boldsymbol{k},s)e^{-ik\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} = \frac{u^a(\boldsymbol{k},s)e^{-iE_{\boldsymbol{k}}t+i\boldsymbol{k}\boldsymbol{x}}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \tag{1}$$

$$V^a_{\boldsymbol{k},s}(x) = \frac{v^a(\boldsymbol{k},s)e^{+ik\cdot x}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} = \frac{v^a(\boldsymbol{k},s)e^{+iE_{\boldsymbol{k}}t-i\boldsymbol{k}\boldsymbol{x}}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \tag{2}$$

$$(f|g) = \int d^3\boldsymbol{x} f^\dagger(x) g(x) \tag{3}$$

$$b(\boldsymbol{k},s) = (U^a_{\boldsymbol{k},s}|\psi^a) \tag{4}$$

$$b^\dagger(\boldsymbol{k},s) = (U^a_{\boldsymbol{k},s}|\psi^a)^\dagger = (\psi^a|U^a_{\boldsymbol{k},s}) \tag{5}$$

$$d^\dagger(\boldsymbol{k},s) = (V^a_{\boldsymbol{k},s}|\psi^a) \tag{6}$$

$$d(\boldsymbol{k},s) = (V^a_{\boldsymbol{k},s}|\psi^a)^\dagger = (\psi^a|V^a_{\boldsymbol{k},s}) \tag{7}$$

$$\{\psi^a(t,\boldsymbol{x}), \psi^\dagger_b(t,\boldsymbol{x'})\} = \delta^a{}_b \delta^3(\boldsymbol{x}-\boldsymbol{x'}) \tag{8}$$

$$\{\psi^a(t,\boldsymbol{x}), \psi^b(t,\boldsymbol{x'})\} = \{\psi^\dagger_a(t,\boldsymbol{x}), \psi^\dagger_b(t,\boldsymbol{x'})\} = 0 \tag{9}$$

$$\{b(\boldsymbol{k},s), b^\dagger(\boldsymbol{k'},s')\} = \{(U^a_{\boldsymbol{k},s}|\psi^a), (\psi^b|U^b_{\boldsymbol{k'},s'})\} \tag{10}$$

$$= \left\{ \int d^3\boldsymbol{x} U^\dagger_{\boldsymbol{k},s,a}(x)\psi^a(x), \int d^3\boldsymbol{x'} \psi^\dagger_b(x') U^b_{\boldsymbol{k'},s'}(x') \right\} \tag{11}$$

$$= \int d^3\boldsymbol{x} \int d^3\boldsymbol{x'} U^\dagger_{\boldsymbol{k},s,a}(x) U^b_{\boldsymbol{k'},s'}(x') \{\psi^a(x), \psi^\dagger_b(x')\} \tag{12}$$

$$= \int d^3\boldsymbol{x} \int d^3\boldsymbol{x'} U^\dagger_{\boldsymbol{k},s,a}(t,\boldsymbol{x}) U^b_{\boldsymbol{k'},s'}(t,\boldsymbol{x'}) \delta^a{}_b \delta^3(\boldsymbol{x}-\boldsymbol{x'}) \tag{13}$$

$$= \int d^3\boldsymbol{x} U^\dagger_{\boldsymbol{k},s,a}(t,\boldsymbol{x}) U^a_{\boldsymbol{k'},s'}(t,\boldsymbol{x}) \tag{14}$$

$$= \int d^3\boldsymbol{x} \frac{u^\dagger_a(\boldsymbol{k},s)e^{+iE_{\boldsymbol{k}}t-i\boldsymbol{k}\boldsymbol{x}}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \frac{u^a(\boldsymbol{k'},s')e^{-iE_{\boldsymbol{k'}}t+i\boldsymbol{k'}\boldsymbol{x}}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} \tag{15}$$

$$= \delta(\boldsymbol{k}-\boldsymbol{k'}) \frac{u^\dagger_a(\boldsymbol{k},s)u^a(\boldsymbol{k},s')}{2E_{\boldsymbol{k}}} \tag{16}$$

$$= \delta(\boldsymbol{k}-\boldsymbol{k'})\delta_{ss'} \tag{17}$$

$$\{d(\boldsymbol{k},s),d^\dagger(\boldsymbol{k'},s')\} = \{(V^a_{\boldsymbol{k},s}|\psi^a),(\psi^b|V^b_{\boldsymbol{k'},s'})\} \tag{18}$$
$$= \left\{\int d^3\boldsymbol{x} V^\dagger_{\boldsymbol{k},s,a}(x)\psi^a(x), \int d^3\boldsymbol{x'}\psi^\dagger_b(x')V^b_{\boldsymbol{k'},s'}(x')\right\} \tag{19}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} V^\dagger_{\boldsymbol{k},s,a}(x)V^b_{\boldsymbol{k'},s'}(x')\{\psi^a(x),\psi^\dagger_b(x')\} \tag{20}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} V^\dagger_{\boldsymbol{k},s,a}(t,\boldsymbol{x})V^b_{\boldsymbol{k'},s'}(t,\boldsymbol{x'})\delta^a{}_b\delta^3(\boldsymbol{x}-\boldsymbol{x'}) \tag{21}$$
$$= \int d^3\boldsymbol{x} V^\dagger_{\boldsymbol{k},s,a}(t,\boldsymbol{x})V^a_{\boldsymbol{k'},s'}(t,\boldsymbol{x}) \tag{22}$$
$$= \int d^3\boldsymbol{x} \frac{v^\dagger_a(\boldsymbol{k},s)e^{-iE_{\boldsymbol{k}}t+i\boldsymbol{kx}}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}}\frac{v^a(\boldsymbol{k'},s')e^{+iE_{\boldsymbol{k'}}t-i\boldsymbol{k'x}}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} \tag{23}$$
$$= \delta(\boldsymbol{k}-\boldsymbol{k'})\frac{v^\dagger_a(\boldsymbol{k},s)v^a(\boldsymbol{k},s')}{2E_{\boldsymbol{k}}} \tag{24}$$
$$= \delta(\boldsymbol{k}-\boldsymbol{k'})\delta_{ss'} \tag{25}$$
$$\{b(\boldsymbol{k},s),d^\dagger(\boldsymbol{k'},s')\} = \{(U^a_{\boldsymbol{k},s}|\psi^a),(\psi^b|V^b_{\boldsymbol{k'},s'})\} \tag{26}$$
$$= \left\{\int d^3\boldsymbol{x} U^\dagger_{\boldsymbol{k},s,a}(x)\psi^a(x), \int d^3\boldsymbol{x'}\psi^\dagger_b(x')V^b_{\boldsymbol{k'},s'}(x')\right\} \tag{27}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} U^\dagger_{\boldsymbol{k},s,a}(x)V^b_{\boldsymbol{k'},s'}(x')\{\psi^a(x),\psi^\dagger_b(x')\} \tag{28}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} U^\dagger_{\boldsymbol{k},s,a}(t,\boldsymbol{x})V^b_{\boldsymbol{k'},s'}(t,\boldsymbol{x'})\delta^a{}_b\delta^3(\boldsymbol{x}-\boldsymbol{x'}) \tag{29}$$
$$= \int d^3\boldsymbol{x} U^\dagger_{\boldsymbol{k},s,a}(t,\boldsymbol{x})V^a_{\boldsymbol{k'},s'}(t,\boldsymbol{x}) \tag{30}$$
$$= \int d^3\boldsymbol{x} \frac{u^\dagger_a(\boldsymbol{k},s)e^{+iE_{\boldsymbol{k}}t-i\boldsymbol{kx}}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}}\frac{v^a(\boldsymbol{k'},s')e^{+iE_{\boldsymbol{k'}}t-i\boldsymbol{k'x}}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} \tag{31}$$
$$= \delta(\boldsymbol{k}+\boldsymbol{k'})e^{+2iE_{\boldsymbol{k}}t}\frac{u^\dagger_a(\boldsymbol{k},s)v^a(-\boldsymbol{k},s')}{2E_{\boldsymbol{k}}} \tag{32}$$
$$= 0 \tag{33}$$
$$\{d(\boldsymbol{k},s),b^\dagger(\boldsymbol{k'},s')\} = \{(V^a_{\boldsymbol{k},s}|\psi^a),(\psi^b|U^b_{\boldsymbol{k'},s'})\} \tag{34}$$
$$= \left\{\int d^3\boldsymbol{x} V^\dagger_{\boldsymbol{k},s,a}(x)\psi^a(x), \int d^3\boldsymbol{x'}\psi^\dagger_b(x')U^b_{\boldsymbol{k'},s'}(x')\right\} \tag{35}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} V^\dagger_{\boldsymbol{k},s,a}(x)U^b_{\boldsymbol{k'},s'}(x')\{\psi^a(x),\psi^\dagger_b(x')\} \tag{36}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} V^\dagger_{\boldsymbol{k},s,a}(t,\boldsymbol{x})U^b_{\boldsymbol{k'},s'}(t,\boldsymbol{x'})\delta^a{}_b\delta^3(\boldsymbol{x}-\boldsymbol{x'}) \tag{37}$$
$$= \int d^3\boldsymbol{x} V^\dagger_{\boldsymbol{k},s,a}(t,\boldsymbol{x})U^a_{\boldsymbol{k'},s'}(t,\boldsymbol{x}) \tag{38}$$
$$= \int d^3\boldsymbol{x} \frac{v^\dagger_a(\boldsymbol{k},s)e^{-iE_{\boldsymbol{k}}t+i\boldsymbol{kx}}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}}\frac{u^a(\boldsymbol{k'},s')e^{-iE_{\boldsymbol{k'}}t+i\boldsymbol{k'x}}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} \tag{39}$$
$$= \delta(\boldsymbol{k}+\boldsymbol{k'})e^{-2iE_{\boldsymbol{k}}t}\frac{v^\dagger_a(\boldsymbol{k},s)u^a(-\boldsymbol{k},s')}{2E_{\boldsymbol{k}}} \tag{40}$$
$$= 0 \tag{41}$$

$$\{b(\boldsymbol{k},s),b(\boldsymbol{k'},s')\} = \{(U^a_{\boldsymbol{k},s}|\psi^a),(U^b_{\boldsymbol{k'},s'}|\psi^b)\} \tag{42}$$
$$= \left\{\int d^3\boldsymbol{x} U^\dagger_{\boldsymbol{k},s,a}(x)\psi^a(x), \int d^3\boldsymbol{x'} U^\dagger_{\boldsymbol{k'},s',b}(x')\psi^b(x')\right\} \tag{43}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} U^\dagger_{\boldsymbol{k},s,a}(x)U^\dagger_{\boldsymbol{k'},s',b}(x')\{\psi^a(x),\psi^b(x')\} \tag{44}$$
$$= 0 \tag{45}$$
$$\{d(\boldsymbol{k},s),d(\boldsymbol{k'},s')\} = \{(V^a_{\boldsymbol{k},s}|\psi^a),(V^b_{\boldsymbol{k'},s'}|\psi^b)\} \tag{46}$$
$$= \left\{\int d^3\boldsymbol{x} V^\dagger_{\boldsymbol{k},s,a}(x)\psi^a(x), \int d^3\boldsymbol{x'} V^\dagger_{\boldsymbol{k'},s',b}(x')\psi^b(x')\right\} \tag{47}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} V^\dagger_{\boldsymbol{k},s,a}(x)V^\dagger_{\boldsymbol{k'},s',b}(x')\{\psi^a(x),\psi^b(x')\} \tag{48}$$
$$= 0 \tag{49}$$
$$\{b(\boldsymbol{k},s),d(\boldsymbol{k'},s')\} = \{(U^a_{\boldsymbol{k},s}|\psi^a),(V^b_{\boldsymbol{k'},s'}|\psi^b)\} \tag{50}$$
$$= \left\{\int d^3\boldsymbol{x} U^\dagger_{\boldsymbol{k},s,a}(x)\psi^a(x), \int d^3\boldsymbol{x'} V^\dagger_{\boldsymbol{k'},s',b}(x')\psi^b(x')\right\} \tag{51}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} U^\dagger_{\boldsymbol{k},s,a}(x)V^\dagger_{\boldsymbol{k'},s',b}(x')\{\psi^a(x),\psi^b(x')\} \tag{52}$$
$$= 0 \tag{53}$$
$$\{b^\dagger(\boldsymbol{k},s),b^\dagger(\boldsymbol{k'},s')\} = \{(\psi^a|U^a_{\boldsymbol{k},s}),(\psi^b|U^b_{\boldsymbol{k'},s'})\} \tag{54}$$
$$= \left\{\int d^3\boldsymbol{x}\psi^\dagger_a(x)U^a_{\boldsymbol{k},s}(x), \int d^3\boldsymbol{x'}\psi^\dagger_b(x')U^b_{\boldsymbol{k'},s'}(x')\right\} \tag{55}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} U^a_{\boldsymbol{k},s}(x)U^b_{\boldsymbol{k'},s'}(x')\{\psi^\dagger_a(x),\psi^\dagger_b(x')\} \tag{56}$$
$$= 0 \tag{57}$$
$$\{d^\dagger(\boldsymbol{k},s),d^\dagger(\boldsymbol{k'},s')\} = \{(\psi^a|V^a_{\boldsymbol{k},s}),(\psi^b|V^b_{\boldsymbol{k'},s'})\} \tag{58}$$
$$= \left\{\int d^3\boldsymbol{x}\psi^\dagger_a(x)V^a_{\boldsymbol{k},s}(x), \int d^3\boldsymbol{x'}\psi^\dagger_b(x')V^b_{\boldsymbol{k'},s'}(x')\right\} \tag{59}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} V^a_{\boldsymbol{k},s}(x)V^b_{\boldsymbol{k'},s'}(x')\{\psi^\dagger_a(x),\psi^\dagger_b(x')\} \tag{60}$$
$$= 0 \tag{61}$$
$$\{b^\dagger(\boldsymbol{k},s),d^\dagger(\boldsymbol{k'},s')\} = \{(\psi^a|U^a_{\boldsymbol{k},s}),(\psi^b|V^b_{\boldsymbol{k'},s'})\} \tag{62}$$
$$= \left\{\int d^3\boldsymbol{x}\psi^\dagger_a(x)U^a_{\boldsymbol{k},s}(x), \int d^3\boldsymbol{x'}\psi^\dagger_b(x')V^b_{\boldsymbol{k'},s'}(x')\right\} \tag{63}$$
$$= \int d^3\boldsymbol{x}\int d^3\boldsymbol{x'} U^a_{\boldsymbol{k},s}(x)V^b_{\boldsymbol{k'},s'}(x')\{\psi^\dagger_a(x),\psi^\dagger_b(x')\} \tag{64}$$
$$= 0 \tag{65}$$

$$\psi^a(t,\boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \sum_{s=\pm\frac{1}{2}} [b(\boldsymbol{k},s)u^a(\boldsymbol{k},s)e^{-iE_{\boldsymbol{k}}t+i\boldsymbol{kx}} + d^\dagger(\boldsymbol{k},s)v^a(\boldsymbol{k},s)e^{+iE_{\boldsymbol{k}}t-i\boldsymbol{kx}}] \tag{66}$$

$$\psi_a^\dagger(t,\boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \sum_{s=\pm\frac{1}{2}} [b^\dagger(\boldsymbol{k},s)u_a^\dagger(\boldsymbol{k},s)e^{+iE_{\boldsymbol{k}}t-i\boldsymbol{kx}} + d(\boldsymbol{k},s)v_a^\dagger(\boldsymbol{k},s)e^{-iE_{\boldsymbol{k}}t+i\boldsymbol{kx}}] \tag{67}$$

$$\begin{aligned}\{\psi^a(t,\boldsymbol{x}),\psi_b^\dagger(t,\boldsymbol{x'})\} = \Bigg\{ &\int \frac{d^3\boldsymbol{k}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \sum_{s=\pm\frac{1}{2}} [b(\boldsymbol{k},s)u^a(\boldsymbol{k},s)e^{-iE_{\boldsymbol{k}}t+i\boldsymbol{kx}} + d^\dagger(\boldsymbol{k},s)v^a(\boldsymbol{k},s)e^{+iE_{\boldsymbol{k}}t-i\boldsymbol{kx}}], \\ &\int \frac{d^3\boldsymbol{k'}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} \sum_{s'=\pm\frac{1}{2}} [b^\dagger(\boldsymbol{k'},s')u_b^\dagger(\boldsymbol{k'},s')e^{+iE_{\boldsymbol{k'}}t-i\boldsymbol{k'x'}} + d(\boldsymbol{k'},s')v_b^\dagger(\boldsymbol{k'},s')e^{-iE_{\boldsymbol{k'}}t+i\boldsymbol{k'x'}}] \Bigg\}\end{aligned} \tag{68}$$

$$\begin{aligned}= \int \frac{d^3\boldsymbol{k}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \sum_{s=\pm\frac{1}{2}} \int \frac{d^3\boldsymbol{k'}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} \sum_{s'=\pm\frac{1}{2}} [\\ u^a(\boldsymbol{k},s)u_b^\dagger(\boldsymbol{k'},s')e^{-iE_{\boldsymbol{k}}t+iE_{\boldsymbol{k'}}t+i\boldsymbol{kx}-i\boldsymbol{k'x'}} \quad \{b(\boldsymbol{k},s),b^\dagger(\boldsymbol{k'},s')\} \\ + v^a(\boldsymbol{k},s)v_b^\dagger(\boldsymbol{k'},s')e^{+iE_{\boldsymbol{k}}t-iE_{\boldsymbol{k'}}t-i\boldsymbol{kx}+i\boldsymbol{k'x'}}\{d^\dagger(\boldsymbol{k},s),d(\boldsymbol{k'},s')\}]\end{aligned} \tag{69}$$

$$\begin{aligned}= \int \frac{d^3\boldsymbol{k}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k}}}} \sum_{s=\pm\frac{1}{2}} \int \frac{d^3\boldsymbol{k'}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{k'}}}} \sum_{s'=\pm\frac{1}{2}} [\\ u^a(\boldsymbol{k},s)u_b^\dagger(\boldsymbol{k'},s')e^{-iE_{\boldsymbol{k}}t+iE_{\boldsymbol{k'}}t+i\boldsymbol{kx}-i\boldsymbol{k'x'}} \quad \delta(\boldsymbol{k}-\boldsymbol{k'})\delta_{ss'} \\ + v^a(\boldsymbol{k},s)v_b^\dagger(\boldsymbol{k'},s')e^{+iE_{\boldsymbol{k}}t-iE_{\boldsymbol{k'}}t-i\boldsymbol{kx}+i\boldsymbol{k'x'}} \quad \delta(\boldsymbol{k}-\boldsymbol{k'})\delta_{ss'}]\end{aligned} \tag{70}$$

$$= \int \frac{d^3\boldsymbol{k}}{(2\pi)^3 2E_{\boldsymbol{k}}} \sum_{s=\pm\frac{1}{2}} [u^a(\boldsymbol{k},s)\bar{u}_b(\boldsymbol{k},s)\gamma^0 e^{+i\boldsymbol{kx}-i\boldsymbol{kx'}} - v^a(-\boldsymbol{k},s)\bar{v}_b(-\boldsymbol{k},s)\gamma^0 e^{+i\boldsymbol{kx}-i\boldsymbol{kx'}}] \tag{71}$$

$$= \int \frac{d^3\boldsymbol{k}}{(2\pi)^3 2E_{\boldsymbol{k}}} [\sum_{s=\pm\frac{1}{2}} u^a(\boldsymbol{k},s)\bar{u}_b(\boldsymbol{k},s) - \sum_{s=\pm\frac{1}{2}} v^a(-\boldsymbol{k},s)\bar{v}_b(-\boldsymbol{k},s)]\gamma^0 e^{+i\boldsymbol{kx}-i\boldsymbol{kx'}} \tag{72}$$

$$= \int \frac{d^3\boldsymbol{k}}{(2\pi)^3 2E_{\boldsymbol{k}}} [\left(\frac{\not{k}+m}{2m}\right)^a_b - \left(\frac{\not{k}-m}{2m}\right)^a_b] 2E_{\boldsymbol{k}} e^{+i\boldsymbol{kx}-i\boldsymbol{kx'}} \tag{73}$$

$$= \delta^a{}_b \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} e^{+i\boldsymbol{kx}-i\boldsymbol{kx'}} \tag{74}$$

$$= \delta^a{}_b \delta(\boldsymbol{x}-\boldsymbol{x'}) \tag{75}$$

$$\{\psi^a(t,\boldsymbol{x}),\psi^b(t,\boldsymbol{x'})\} = \{\psi_a^\dagger(t,\boldsymbol{x}),\psi_b^\dagger(t,\boldsymbol{x'})\} = 0 \tag{76}$$

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)