場の量子論 p372check12.9

谷脇 貞善

2025 年 6 月 6 日

1　本文

$$H = -\omega d d^\dagger \qquad (\omega > 0) \tag{1}$$
$$[d, d^\dagger] = 1 \tag{2}$$

(I)

$$d|0\rangle = 0 \qquad (\langle 0|0\rangle = 1) \tag{3}$$
$$|n\rangle = \frac{(d^\dagger)^n}{\sqrt{n!}}|0\rangle \tag{4}$$
$$H|n\rangle = \frac{-\omega d (d^\dagger)^{n+1}}{\sqrt{n!}}|0\rangle \tag{5}$$
$$= -\omega(n+1)[d, d^\dagger]|n\rangle \tag{6}$$
$$= -\omega(n+1)|n\rangle \tag{7}$$

(II)

$$d^\dagger|\tilde{0}\rangle = 0 \qquad (\langle\tilde{0}|\tilde{0}\rangle = 1) \tag{8}$$
$$|\tilde{n}\rangle = \frac{d^{\tilde{n}}}{\sqrt{\tilde{n}!}}|\tilde{0}\rangle \tag{9}$$
$$H|\tilde{n}\rangle = \frac{-\omega d d^\dagger d^{\tilde{n}}}{\sqrt{\tilde{n}!}}|\tilde{0}\rangle \tag{10}$$
$$= -\omega\tilde{n}[d^\dagger, d]|\tilde{n}\rangle \tag{11}$$
$$= \omega\tilde{n}|\tilde{n}\rangle \tag{12}$$
$$\langle\tilde{n}|\tilde{n}\rangle = \langle\tilde{0}|\frac{(d^\dagger)^{\tilde{n}} d^{\tilde{n}}}{\tilde{n}!}|\tilde{0}\rangle \tag{13}$$
$$= [d^\dagger, d]^{\tilde{n}} \tag{14}$$
$$= (-1)^{\tilde{n}} \tag{15}$$

参考文献

[1] 坂本眞人著「場の量子論-不変性と自由場を中心にして」(裳華房,2014,ISBN9784785325114)